Taylor's Formula with Remainder.

If $f(x)$ is $(n+1)$-times differentiable in $(a-r, a+r)$ and $f^{(n+1)}(x)$ is continuous, then for each x in $(a-r, a+r)$ there exists a number c in $(a-r, a+r)$ for which the equation

$$f(x) = \underbrace{\sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-a)^k}_{\text{Polynomial of degree } n} + \underbrace{\frac{1}{(n+1)!} f^{(n+1)}(c)(x-a)^{n+1}}_{\text{Remainder}}$$

Examples: $\quad e^x = \sum_{0}^{\infty} \frac{x^k}{k!} \qquad |x| < \infty$

$f^{(k)}(a)$

odd $\quad \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \qquad |x| < \infty$

even $\quad \cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!} \qquad |x| < \infty$