

If there is a positive number r so that T_N

$$\lim_{N \rightarrow \infty} \sum_{k=0}^N \frac{f^{(k)}(a)}{k!} (x-a)^k = f(x)$$



for $a-r < x < a+r$,

then f is called analytic at the point $x=a$.

Then the infinite series

$$\sum_{k=0}^{\infty} \left(\frac{f^{(k)}(a)}{k!} (x-a)^k \right)$$

is called Taylor's Series for the function f centered at $x=a$.

In the special case that $a=0$, the series becomes

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k,$$

which is called a Maclaurin Series.

$$f(x) = e^x \quad f^{(k)}(x) = e^x \quad f^{(k)}(0) = 1$$
$$\sum_{k=0}^{\infty} \frac{1}{k!} x^k = e^x$$