Taylor's Coefficients, Taylor's Polynomials, and Taylor's Series.

If $f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n$, for $a-r < x < a+r$

then $f^{(k)}(x) = \sum_{n=k}^{\infty} n(n-1)\cdots(n-k+1) C_n (x-a)^{n-k}$

$$\Rightarrow \quad f^{(k)}(a) = \underbrace{k(k-1)\cdots(2)(1)}_{k!} C_k = k! C_k$$

$$\Rightarrow \quad \boxed{C_k = \frac{f^{(k)}(a)}{k!}}$$ Taylor's coefficient formula.

If f has N derivatives at a point a, then

$$T_N(x) = \sum_{k=0}^{(N)} \frac{f^{(k)}(a)}{k!} (x-a)^k$$

is called Taylor's Polynomial for f of degree N centered at $x = a$.