II. Sometimes an exact expression for the n^{th} partial sums is not so easy, but we can <u>estimate</u> S_n and that's good enough.

<u>Example 3.</u> $\sum_{1}^{\infty} \frac{1}{n^2}$ converges.

Let $S_n = 1 + \frac{1}{2^2} + \cdots + \frac{1}{n^2} < 1 + \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = 2 - \frac{1}{n+1} < 2.$

$\{S_n\}$ is increasing (since all the terms are positive) and the sums S_n are bounded above by 2, so $\lim_{n\to\infty} S_n$ exists.

(The exact value happens to be $\pi^2/6$ and you can check this on your TI-92 with the "SUM" command.)

<u>Example 4.</u> $\sum_{1}^{\infty} \frac{1}{n}$ diverges (to $+\infty$), even though the individual terms $1/n$ go to 0. Here is one way to see it:

$$S_{2^n} = 1 + \left(\tfrac{1}{2}\right) + \overset{>\frac{1}{2}}{\left(\tfrac{1}{3} + \tfrac{1}{4}\right)} + \overset{>\frac{1}{2}}{\left(\tfrac{1}{5} + \cdots + \tfrac{1}{8}\right)} + \overset{>\frac{1}{2}}{\left(\tfrac{1}{9} + \cdots + \tfrac{1}{16}\right)} + \cdots + \overset{>\frac{1}{2}}{\left(\tfrac{1}{2^{n-1}+1} + \cdots + \tfrac{1}{2^n}\right)}$$

$$\underline{\geq 1 + \tfrac{n}{2}} \to +\infty \text{ as } n \to +\infty.$$