Geometric Series Formula:

$$\sum_{1}^{\infty} \frac{1}{2^{n-1}} = 2$$

If $|r|<1$, then $\displaystyle\sum_{1}^{\infty} cr^{k-1} = \frac{c}{1-r}$.

If $|r|\geq 1$, then $\displaystyle\sum_{1}^{\infty} cr^{n-1}$ diverges (unless $c=0$).

Meaning: $\displaystyle\lim_{n\to\infty}\Big[\underbrace{\sum_{k=1}^{n} cr^{k-1}}_{S_n} - \frac{c}{1-r}\Big] = \lim_{n\to\infty} \frac{cr^n}{1-r} = 0$ if

$\underline{-1 < r < +1}$

Example 2. $\displaystyle\sum_{1}^{\infty} \frac{1}{n(n+1)} = 1$ (telescoping sum).

Use the fact (partial fraction decomposition) that

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}.$$

So $S_n = \frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{n(n+1)}$

$$= \left(1 - \cancel{\tfrac{1}{2}}\right) + \left(\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}}\right) + \left(\cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}}\right) + \cancel{\cdots} + \left(\cancel{\tfrac{1}{n-1}} - \cancel{\tfrac{1}{n}}\right) + \left(\cancel{\tfrac{1}{n}} - \tfrac{1}{n+1}\right)$$

$$= 1 - \frac{1}{n+1} \to 1 \quad \text{as } n\to\infty.$$