

Given a sequence  $\{a_n\}$ , how can we tell whether or not the infinite series  $\sum_1^{\infty} a_n$  converges?

**I:** Find a formula for  $S_n$  as a single expression (in closed form) and see if  $\lim_{n \rightarrow \infty} S_n$  exists.

Example 1.  $a_n = c r^{n-1}$ , where  $c \neq 0$  is a constant (independent of  $n$ )

and  $r$  is a fixed real number (ratio) that is not equal to 1.

This is called a geometric sequence and the ratio

between consecutive terms is  $\frac{a_{n+1}}{a_n} = \frac{c r^{n+1}}{c r^n} = r$ .

Geometric Sum Formula: For any  $r \neq 1$ , and any  $c$

$$\sum_1^n c r^{k-1} = \frac{c(r^n - 1)}{r - 1} = \frac{c(1 - r^n)}{1 - r}$$

If  $|r| < 1$ ,  $\lim_{n \rightarrow \infty} r^n = 0$ ; otherwise  $\lim_{n \rightarrow \infty} r^n$  DNE.