Infinite Series are in many ways like improper integrals of the first kind. Remember, $\int_a^{+\infty} f(x)\,dx = \lim_{b\to+\infty} \int_a^b f(x)\,dx$

if the limit exists and we use the same term "convergence" for this limiting process.

n^{th} term test

A necessary condition for $\sum_{n=1}^{\infty} a_n$ to converge (to a sum)

is that $\boxed{\lim_{n\to\infty} a_n = 0}$ This is because $S_n = S_{n-1} + a_n$

and $S_n \to S$ as $n\to\infty$ and $S_{n-1} \to S$, too, as $n\to\infty$.

But if $\lim_{n\to\infty} a_n = 0$, it does not necessarily imply that $\sum_1^{\infty} a_n$ converges. The convergence or divergence of the series depends in some sense both on the speed or rate at which the terms are approaching 0 as well as any cancellation that may take place caused by alternating signs.