Infinite Series of Numbers.

Suppose we are given an infinite sequence of numbers

$\{a_n\}_{n=1}^{\infty}$. Form another sequence by taking sums:

terms

$$
\left\{
\begin{aligned}
S_1 &= a_1 \\
S_2 &= a_1 + a_2 \\
S_3 &= a_1 + a_2 + a_3 \\
&\vdots \\
S_n &= a_1 + a_2 + \cdots + a_n \quad n^{\text{th}} \text{ partial sum} \\
&\vdots
\end{aligned}
\right.
$$

The sequence $\{S_n\}$ is called the sequence of partial sums

of the "infinite" series $\sum_1^{\infty} a_n = S.$

$a_1 + a_2 + \cdots + a_n + \cdots$

If $\boxed{\lim_{n\to\infty} S_n = S}$ exists (and is a finite real number), we

say the infinite series $\sum_{n=1}^{\infty} a_n$ converges to the sum S.

If $\lim_{n\to\infty} S_n$ DNE or is ∞, we say $\sum_1^{\infty} a_n$ diverges.