PROPERTIES OF FUNCTIONS GIVEN BY POWER SERIES

$$f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1(x-a) + c_2(x-a)^2 + \dots$$

$$(a-r, a+r)$$

Assume the power series converges for $|x-a| < r$.

Then

(1) $f(x)$ is <u>differentiable</u> in $(a-r, a+r)$ and

$$\frac{d}{dx}(x-a)^n = n(x-a)^{n-1} \qquad f'(x) = \sum_{n=1}^{\infty} n c_n (x-a)^{n-1} = c_1 + 2c_2(x-a) + \dots$$

Also $f''(x)$ exists and

$$f''(x) = \sum_{n=2}^{\infty} n(n-1) c_n (x-a)^{n-2} = 2c_2 + 6c_3(x-a) + \dots$$

etc.... <u>All</u> derivatives exist and

$$\frac{d^k f}{dx^k} = f^{(k)}(x) = \sum_{n=k}^{\infty} n(n-1)\cdots(n-k+1) c_n (x-a)^{n-k}$$

$$= \underbrace{k(k-1)(k-2)\cdots(2)(1)}_{k!} c_k + \dots$$

$$\underbrace{k(k-1)(k-2)\cdots(2)(1)}_{k!}$$