

The largest value  $r \geq 0$  for which the power series

converges for all  $x$  satisfying  $|x-a| < r$   $-r < x-a < r$

and diverges for all  $x$  satisfying  $|x-a| > r$

is called the radius of convergence of the power series.

At the end points  $x = a+r$  and  $x = a-r$ , the power series may or may not converge.

Examples:

$$\sum_0^{\infty} \frac{(x-2)^n}{3^n} = 1 + \frac{x-2}{3} + \frac{(x-2)^2}{9} + \dots$$

$$a_n = \frac{(x-2)^n}{3^n} \quad a_{n+1} = \frac{(x-2)^{n+1}}{3^{n+1}}$$

$$\left| \frac{a_{n+1}}{a_n} \right| = \left| \frac{(x-2)^{n+1} 3^n}{3^{n+1} (x-2)^n} \right| = \left| \frac{x-2}{3} \right|$$

Ratio Test:  $\left| \frac{x-2}{3} \right| < 1 \Rightarrow$  Convergence.

$r=3$ .  $\left| \frac{x-2}{3} \right| > 1 \Rightarrow$  divergence

$$-3 < x-2 < 3$$

diverges  $\downarrow$  Convergence  $\uparrow$  diverges  
 $a-r$   $a$   $a+r$