

Examples of L'Hospital's Rule for sequences.

$$\lim_{n \rightarrow \infty} \frac{n}{e^n} = \lim_{x \rightarrow \infty} \frac{x}{e^x} = \lim_{x \rightarrow \infty} \frac{1}{e^x} = 0$$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n} = \lim_{x \rightarrow \infty} \frac{\ln x}{x} = \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$$

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = ? \quad \text{Let } a_n = \ln\left(1 + \frac{1}{n}\right)^n = n \ln\left(1 + \frac{1}{n}\right)$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} n \ln\left(1 + \frac{1}{n}\right) = \lim_{n \rightarrow \infty} \frac{\ln\left(1 + \frac{1}{n}\right)}{\frac{1}{n}}$$

$$= \lim_{x \rightarrow \infty} \frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{\frac{1}{1 + \frac{1}{x}} \left(-\frac{1}{x^2}\right)}{\left(-\frac{1}{x^2}\right)} = 1$$

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = 1 \quad \Rightarrow \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{n \rightarrow \infty} e^{a_n} = e^1 = e.$$