

Chapter 12. Infinite Sequences & Series

A sequence is a collection of numbers $\{a_1, a_2, \dots, a_n, \dots\}$ which are indexed by the natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$.

For short, we denote a sequence by $\{a_n\}_{n=1}^{\infty}$.

The terms or values of a sequence can be given in

various ways:

(1) an explicit formula for a_n :

$$a_n = n \quad b_n = \frac{1}{n} \quad c_n = \frac{n^2 + 2n - 3}{n^2 + 4n + 5}$$

$$d_n = \left(1 + \frac{1}{n}\right)^n \quad f_n = \frac{1}{2}n$$

(2) a recursive formula or recurrence relation for a_n :

$$a_{n+2} = a_n + a_{n+1} \quad b_{n+1} = b_n + \frac{1}{n}$$

$$a_1 = 1, a_2 = 1 \quad b_1 = 1$$

(3) a descriptive procedure for calculating each term:

$$f(x) = \tan x \quad a_n = f^{(n)}(0) \quad n \in \mathbb{N}$$

$b_n = n^{\text{th}}$ digit in decimal expansion of $\sqrt{\pi} = 3.14159\dots$