

In Cartesian (x, y) coordinates, the conic sections are exactly those curves which satisfy an equation of the form: General quadratic in x and y

$$\boxed{ax^2 + bxy + cy^2 + dx + ey + f = 0.}$$

To see which kind of conic section you get, calculate $b^2 - 4ac$ which is called the discriminant of the equation.

If $b^2 - 4ac < 0$, you get an ellipse.

If $b^2 - 4ac = 0$, you get a parabola.

If $b^2 - 4ac > 0$, you get a hyperbola.

Ex: $y = x^2$ $b^2 - 4ac = 0$

$$x^2 + y^2 = 1 \quad b^2 - 4ac = -1$$

$$xy = 1 \quad b^2 - 4ac = 1$$

