

Important and Useful Fact:

Any combination of the form $a \cos \theta + b \sin \theta$

can be rewritten in the form $\pm A \cos(\theta - \phi)$, where we choose $+$ if $a > 0$ and $-$ if $a < 0$.

To find A (amplitude) and ϕ (phase shift), expand

$$A \cos(\theta - \phi) = \underline{A \cos \theta \cos \phi + A \sin \theta \sin \phi}$$

$$\Rightarrow \boxed{a = A \cos \phi} \text{ and } \boxed{b = A \sin \phi}$$

$$\Rightarrow a^2 + b^2 = A^2(\cos^2 \phi + \sin^2 \phi) = A^2 \Rightarrow \sqrt{A^2} = \sqrt{a^2 + b^2} = A.$$

$$\text{From } \frac{b}{a} = \frac{A \sin \phi}{A \cos \phi} = \tan \phi \Rightarrow \boxed{\phi = \arctan(b/a)}$$

Where we select \arctan in the interval $[-\pi/2, \pi/2]$.

Example: $r = \frac{2}{3 + 2 \cos \theta - 3 \sin \theta}$; What and where is it?