# FIRST ORDER, LINEAR, DIFFERENTIAL EQUATIONS

have the form

$$(*) \qquad y' = a(x)\,y + b(x)$$

, where $a(x)$ & $b(x)$ are given functions.

Special sub-cases: (1) $a(x) \equiv 0$. $y(x) = \int b(x)\,dx$.

(2) $b(x) \equiv 0$. $y' = a(x)y$ is separable

$$\Rightarrow \frac{dy}{y} = a(x)\,dx \Rightarrow \ln|y| = \int a(x)\,dx \Rightarrow y(x) = \exp\left[\int a(x)\,dx\right].$$

General case: $y' = a(x)y + b(x)$ is not separable.

integrating factor

Let $m(x) = \exp\left[-\int a(x)\,dx\right]$. Then $\boxed{m'(x) = -a(x)\,m(x).}$

Multiply $(*)$ by $m(x)$:

$$\underbrace{\left[m(x)y'(x) - m(x)a(x)y(x)\right]}_{\frac{d}{dx}(m(x)y(x))} = m(x)b(x)$$

Product Rule

$$m(x)y(x) = \int m(x)b(x)\,dx + C$$

$$\boxed{y(x) = \frac{1}{m(x)}\int m(x)b(x)\,dx + \frac{C}{m(x)}}$$