Statistical Image Restoration via the Ising Model

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Math 336: Image Processing

Final Presentation

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An Overview

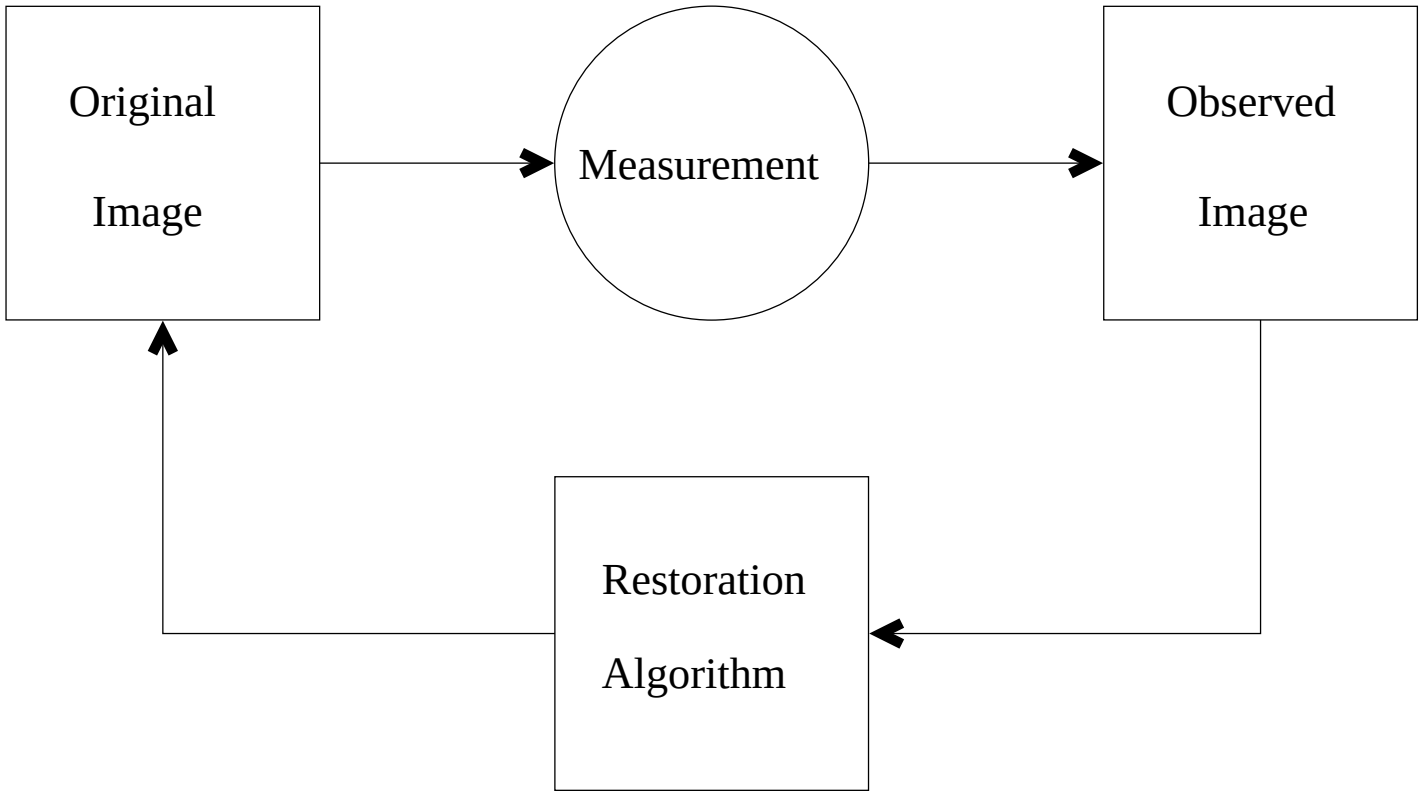
Statistical Image Restoration

Two-Dimensional Ising Model

Binary Image Restoration

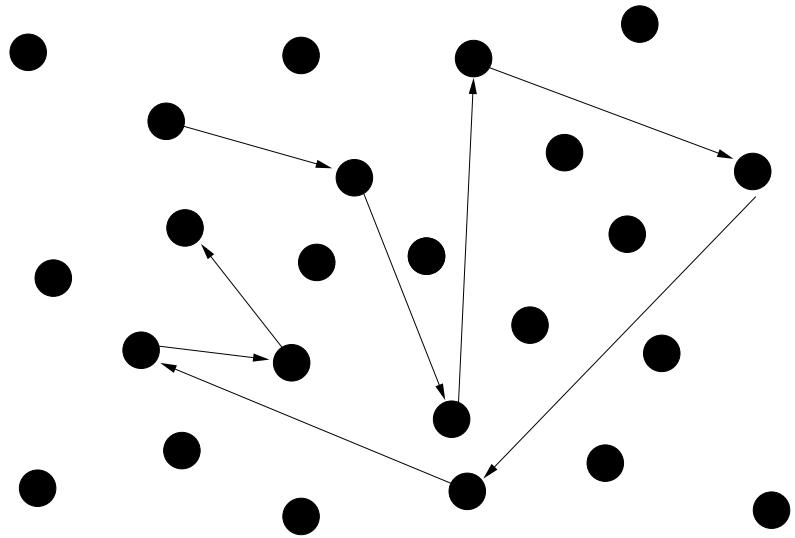
Q-Color Image Restoration

Image Restoration



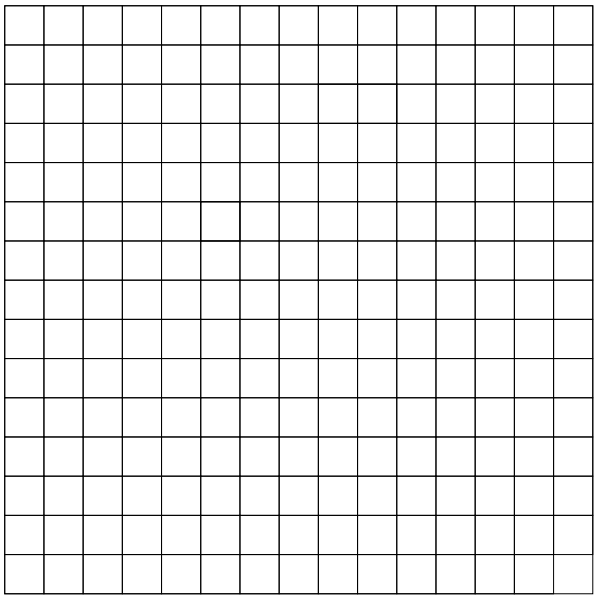
$$G(i,j) = N_M(i,j) \times F(i,j) + N_A(i,j)$$

Statistical Image Restoration



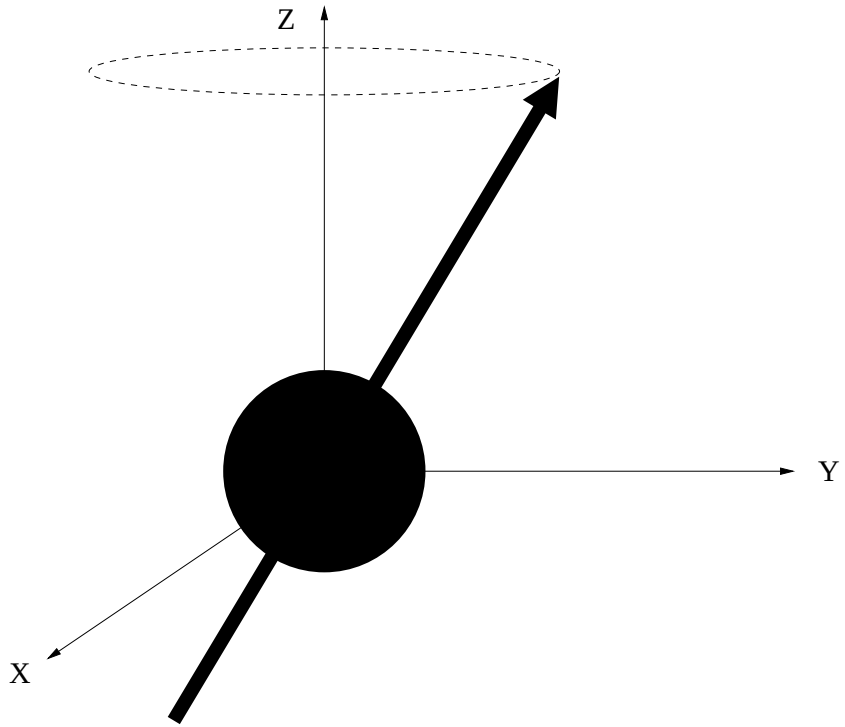
- Estimate probability distribution of the original image.
- Sample phase space via a stochastic process using the estimated probability distribution.
- Image is restored when some configuration condition is met.

Image Representation



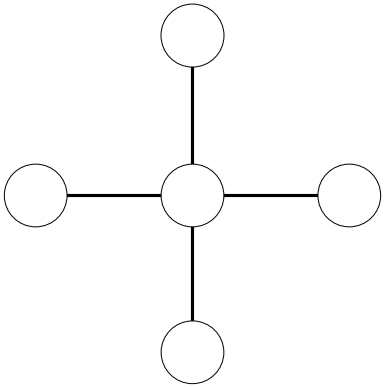
- Indexed Image $[0, 1, 2, \cdots, 255]$
- Grayscale Image $[0.000, 0.001, 0.002, \cdots, 1.000]$
- Binary Image $[0, 1]$

Spin Equivalent Representation



- Intrinsic Angular Momentum
- Quantized
- Magnetic Moment

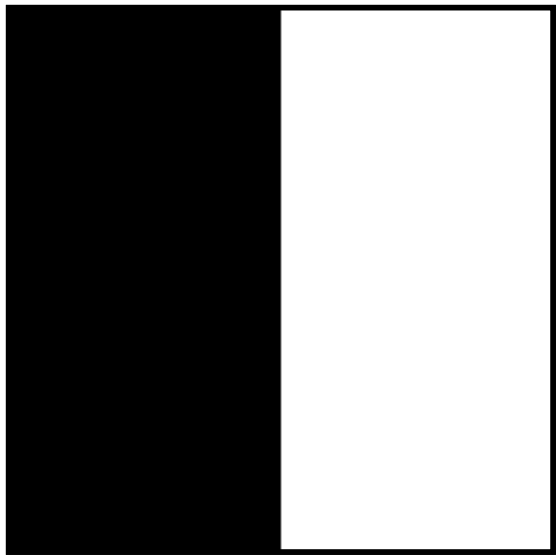
Two-Dimensional Ising Model



$$E = -\mu \sum_{ij} h_{ij} - J \sum_{ijkl} s_{ij} s_{kl}$$

- Simple model of magnetic materials.
- Phase transitions. Spontaneous magnetization.
- Spin up $(+1)$ or spin down (-1) only.
- Spins coupled to an external magnetic field h.
- Spin-spin nearest-neighbor interaction.

Ising Model: A Simple Example



Initial Lattice Configuration

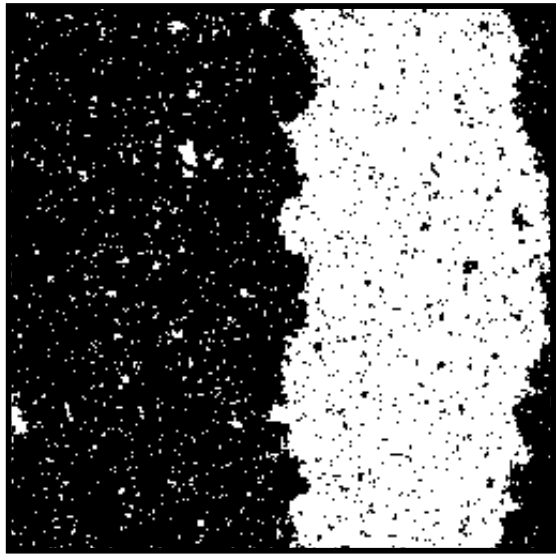
A Simple Example: Zero Field

$$T << T_c$$

A Simple Example: Zero Field

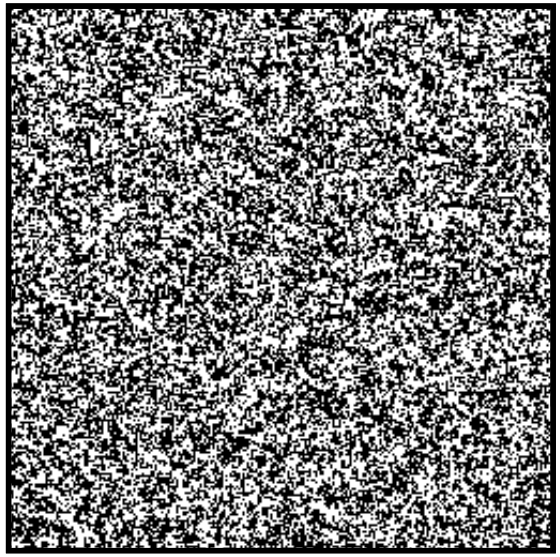
$$T < T_c$$

A Simple Example: Zero Field



$$T \approx T_c$$

A Simple Example: Zero Field



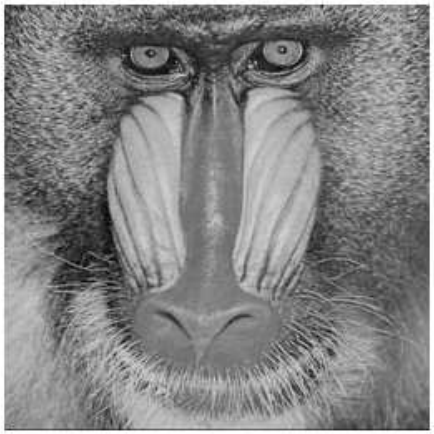
$$T > T_c$$

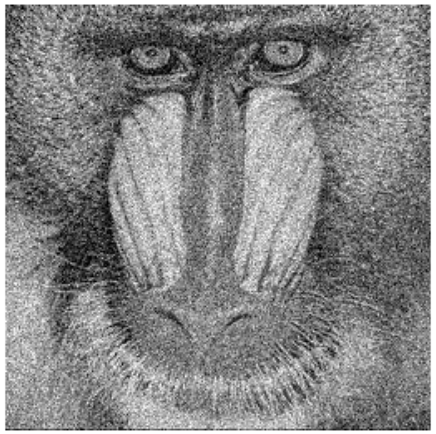
Metropolis Algorithm

Monte Carlo search through phase space to find an energtically favorable configuration of the spin lattice for a given temperature β.

- Choose a spin on the lattice at random.
- Flip the spin. Spin up to spin down. Spin down to spin up.
- Compute energy difference $\Delta E = E' - E$.
- If $\Delta E \leq 0$, then keep new configuration.
- If $\Delta E > 0$, then draw a random number R.
- Accept new configuration iff $e^{-\beta \Delta E} \geq R$.
- Else keep old configuration and repeat for next random spin.

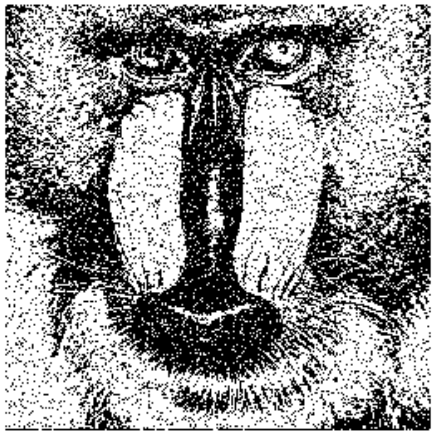
Image Degradation





Binary Image Restoration

Binary Image Restoration

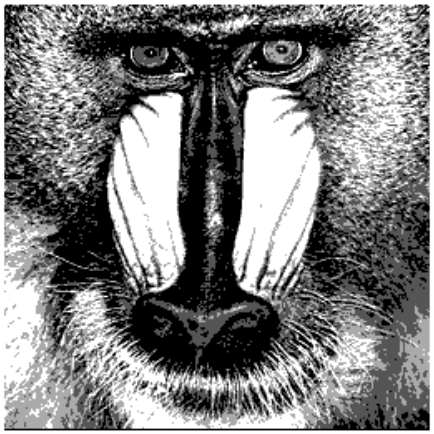


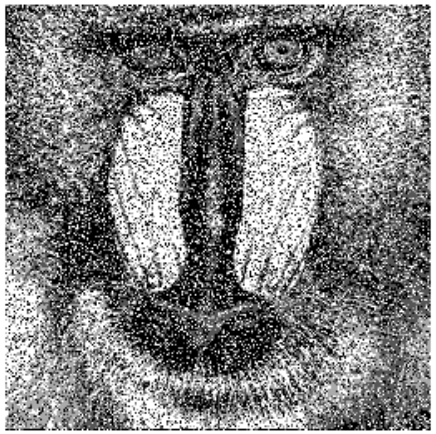
Two-Dimensional Q-Ising Model

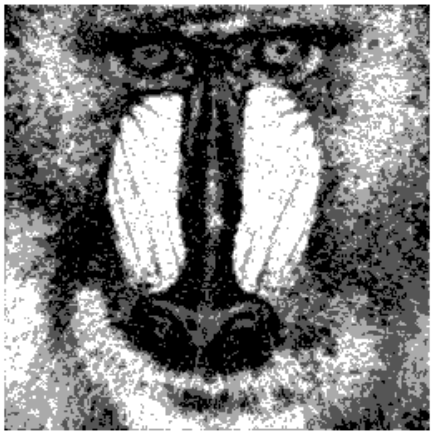
- Generalization of the binary Ising model.
- Used in the study of spin glasses.
- Statistical image restoration of grayscale and color images.

4-Color Image Restoration

4-Color Image Restoration







The End

Thanks!