Computer Labs for Calculus in the Life Sciences with WeBWorK

Society of Math Biology - 2012

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Calculus for the Life Sciences at SDSU

- Calculus I (Math 121) has 150-250 students/semester
- Calculus II (Math 122) has 75-120 students/semester
- Courses – 2 hr Lectures and a 2 hr Computer Lab
  - Wide diversity of preparation amongst students
  - 20-50 students in Labs
- WeBWorK used for Homework and Labs
  - Allows easy management of large classes
  - Improved exam performance 10%
  - Better understanding of homework and labs (individualized)
  - Open source through MAA – instructor controlled

Modeling and Dynamical Systems Approach

- Introduce modeling methods and review functions
- Discrete dynamical models
  - Malthusian growth – Easily accessible
  - Linear discrete models – Stability later connects to derivative
  - Growth provides biological introduction to derivative
- Derivatives
  - Fit models and find extrema
  - Optimization
  - Nonlinear discrete dynamical models - Stability analysis
- Differential equations
  - Linear differential equations - Derive from discrete models
  - Solving differential equations with integrals
  - Some qualitative analysis of differential equations and systems, like competition models

Course Design

- Each topic introduced with **biological application**
  - Crickets chirping and temperature introduce lines
  - U. S. census data explores discrete models
  - Prozac/drugs introduce exponentials
  - Mercury build up in fish described for antiderivatives
- **Computer Lab**
  - Most problems based on real data
  - Use Excel and some Maple to solve – easily adaptable to other computer tools, like MatLab
  - Learn writing and graphing skills
- Many students return to say Lab skills were invaluable in upper division biology courses

Computer Labs

Computer Labs

- Unique experience for students
- Hands on work with computers and math models
- WeBWorK imposes accuracy, so scientific discipline
- Writing and graphing skills improve over the semester
- Students learn to appreciate the value of mathematics in their field of interest
- Work 2–3 problems each week
- Have developed over 70 Lab problems with over 60 adapted to WeBWorK (available from my web site)

Typical Lab Presentation

Typical Lab Session

- Organize students - Random pairings
- Discuss material from previous lab
  - Relate important ideas students should understand
  - Discuss modeling and writing problems observed
- Brief over view of Lab problems (2 - 3)
  - Present key biological ideas (theory) in problems
  - Indicate key mathematical concepts required
- Main presentation
  - Show details of Excel or Maple for managing a similar problem
  - Discuss what is expected in the students' reports
  - Point to important concepts for study
  - Wander lab and answer individual problems

Lab Problem – Beetle Population

Computer Lab Problem: **Beetle Population Growth**

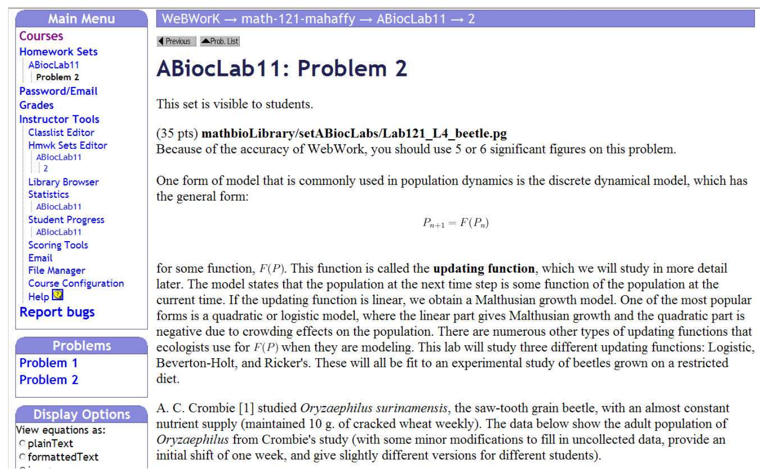
- Students given data (slightly randomized) from study of grain beetles growing with limited resources
- Examine a series of models of the form

$$P_{n+1} = F(P_n)$$

- Use tools in Excel to fit **updating functions** from logistic, Beverton-Holt, and Ricker's models
- Study dynamical systems properties of the models – equilibria and stability
- Study properties of functions – extrema, asymptotes
- Examine time series of different models

Calculus for the Life Sciences

Computer Lab Problem: **Beetle Population Growth**



Computer Lab Problem: Beetle Population Growth

Week	Adults	Week	Adults
0	2	16	406
2	5	18	470
4	28	20	428
6	59	22	426
8	144	24	421
10	293	26	466
12	353	28	436
14	356	30	485

a. The discrete logistic growth model for the adult population P_n can be written

$$P_{n+1} = f(P_n) = rP_n - mP_n^2,$$

where the constants r and m must be determined from the data.

Begin by plotting P_{n+1} vs. P_n, which you can do by entering the adult population data from times 2-30 for P_{n+1} and times 0-28 for P_n. (Be sure that P_n is on the horizontal axis.) To find the appropriate constants use Excel's trendline with its polynomial fit of order 2 and with the intercept set to 0 (under options). Give the best values of the constants, r and m, and write the equation of the model, which fits the data best. Also, find the sum of square errors between the data and this model.

$r =$ ☐
$m =$ ☐

Computer Lab Problem: Beetle Population Growth

b. Another important model used for population dynamics is the Beverton-Holt model, which is given by

$$P_{n+1} = B(P_n) = \frac{aP_n}{1 + \frac{P_n}{b}},$$

where the constants a and b must be determined from the data. Give the best values of the constants, a and b, and write the equation of the model, which fits the data best. (You can make an initial guess of $a = 3$ and $b = 200$.) Also, find the sum of square errors between the data and this model.

$a =$ ☐
$b =$ ☐
$B(P) =$ ☐
$SSE =$ ☐

Find all equilibria for this model (with $P_{1e} < P_{2e}$). Note that equilibria are found by solving

$$P_e = B(P_e)$$

The largest equilibrium represents the carrying capacity of the population.

$P_{1e} =$ ☐
$P_{2e} =$ ☐

Find the derivative of the Beverton-Holt updating function, then find the value of the derivative at all equilibria and give the stability of the equilibria.

$B'(P) =$ ☐

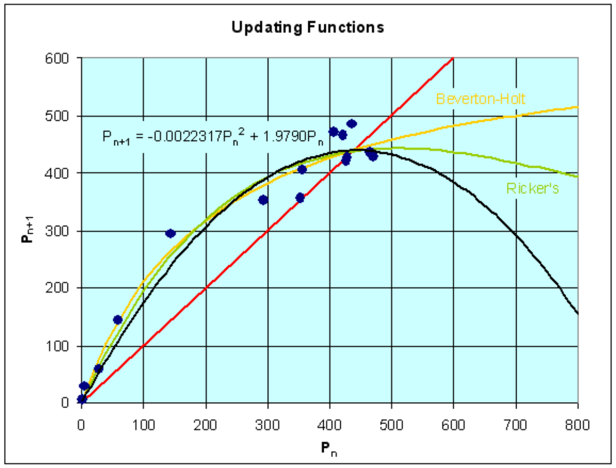
$B'(P_{1e}) =$ ☐ STABLE or UNSTABLE ☐
$B'(P_{2e}) =$ ☐ STABLE or UNSTABLE ☐

c. Another model used for population dynamics is Ricker's model, which is more often used for fish populations and is given by

$$P_{n+1} = R(P_n) = aP_n e^{-\frac{P_n}{b}},$$

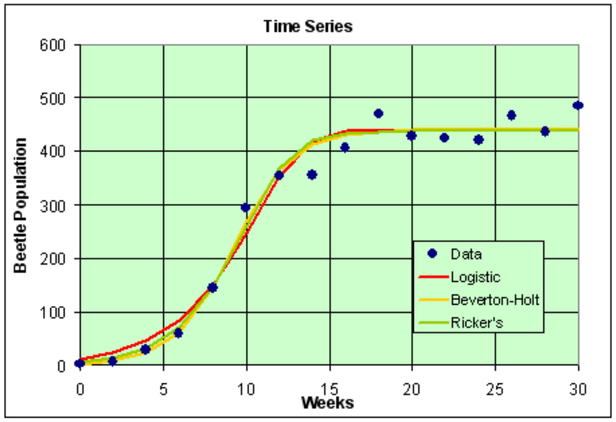
Computer Lab Problem: Beetle Population Growth

Will show WeBWorK problem and demonstrate some of Excel to solve this problem



Computer Lab Problem: Beetle Population Growth

Despite wide variation in model form, time series results are similar



Learning Objectives from Beetle Problem

- Updating Functions
  - Various forms yield similar fits over data
  - Computer algorithms easily fit curves
  - Describe different functional forms – characterize properties
- Time series
  - Discuss different functional relationships
  - Models have very similar behavior
- Modeling – Implications of different forms

Math 121 Computer Lab Problems

Lab 1 (Help page)

1. Lines and Quadratic (A1). Introduction to using Excel for editing graphs and Word for writing equations.
2. Intersection of Line and Quadratic (A2). Graphing a line and a quadratic and finding significant points on the graph.
3. Cricket Thermometer (A3). Listening to crickets on the web, then using a linear model for relating to temperature.

Lab 2 (Help page)

1. Lines and Quadratic (C1). Introduction to Maple for solving equations.
2. Concentration and Absorbance (B2). Linear model for urea concentration measured in a spectrophotometer. Relate to animal physiology.
3. Weak Acids (C2). Solving for [H+] with the quadratic formula, then graphing [H+] and pH.

Lab 3 (Help page)

1. Rational Function and Line (D1). Graphing and finding points of intersection, asymptotes, and intercepts.
2. Growth of Yeast (C3). Linear model for the early growth of a yeast culture. Quadratic to study the least squares best fit.
3. Dog Study (D3). Use an allometric model to study the relationship between length, weight, and surface area of several dogs.

Lab 4 (Help page)

1. Exponential, Logarithm, and Power Functions (E1). Study the relative size of these functions. Finding points of intersection.
2. Island Biodiversity (E2). Fit an allometric model through data on herpetofauna on Caribbean islands.
3. Allegheny Forest (E3). Model volume of trees as a function of diameter or height. Compare linear and allometric models.

Lab 5 (Help page)

1. Malthusian Growth (F2). Data for two countries presented with a discrete Malthusian growth model used for analysis.
2. Malthusian Growth Model for the U. S (F1). Java applet used to find the least squares best fit of growth rate over different intervals of history. Model compared to census data.
3. Malthusian Growth and Nonautonomous Growth Models (F4). Census data analyzed for trends in their growth rates. Models are compared and contrasted to data, then used to project future populations.

Math 121 Computer Lab Problems (cont)

1. Model for Breathing (G2). Examine a linear discrete model for determining vital lung functions for normal and diseased subjects following breathing an enriched source of argon gas.
2. U. S. Census models (H3). The population of the U. S. in the twentieth century is fit with a discrete Malthusian growth model, a Malthusian growth model with immigration, and a logistic growth model. These models are compared for accuracy and used to project future behavior of the population.
3. Weight and Height of Girls (I2). Data on the growth of girls is presented. Allometric modeling compares the relationship between height and weight, then a growth curve is created.

Lab 7 (Help page)

1. Tangent Lines and Derivative (J1). Secant lines are used, then the limit gives the tangent line. Rules of differentiation are explored.
2. Flight of a Ball. Data for a vertically thrown ball is fit, then analyzed (I1). Average velocities are computed for insight into the understanding of the derivative.
3. Growth of Fish (I4). Use von Bertalanffy's equation for estimating the length of fish with some fish data to find growth in length of a fish.

Lab 8 (Help page)

1. Oxygen consumption of *Triatoma phyllosoma* (J2). Cubic polynomial is fit to data for oxygen consumption of this bug. The minimum and maximum are found.
2. Circadian Body Temperature (J4). A cubic polynomial is fit to data for human body temperature as it varies over a 24 hour period. A maximum and minimum are found.
3. Plankton in the Salton Sea (J3). The logarithm of the populations are found, then fit with a quartic polynomials. Extrema are found to find peak populations.

Lab 9 (Help page)

1. Female Body Temperature (J5). A cubic polynomial is fit to data on the female body temperature over one month. Timing of ovulation is related to points of inflection, and the maximum and minimum temperatures are found.
2. Drug Therapy (K3). Models comparing the differences between drug therapies. One case considers injection of the drug, while the other considers slow time release from a polymer.
3. Radioactive Isotopes (K6). Certain radioactive isotopes are used for medical imaging. Exponential function are used to study the decay of these isotopes. The derivative is used to find a maximum and point of inflection.

Math 122 Computer Lab Problems

Lab 1 (Help Page)

1. Continuous Yeast Growth (L2). Data are fit for a growing culture of yeast. Derivatives are used to find the maximum growth in the population.
2. Growth of Pacific Fish (L1). The von Bertalanffy equation is used to find the length of Pacific fish, then an allometric model relates the length to the weight. The chain rule of differentiation is used to find the maximum weight gain as a function of age.

Lab 2 (Help Page)

1. Discrete Models for Birds (L2). Discrete models for the growth of a population of birds is studied. The models that are compared are the logistic growth model, logistic growth model with emigration, and a cubic model with the Allee effect.
2. SIR Model for Influenza (L3). A discrete dynamical system with susceptible and infected individuals is compared to CDC data for the spread of influenza. The model is used to examine different strategies to lessen the effect of the disease.

Lab 3 (Help Page)

1. Optimal Volume (A1). A box is formed from a rectangular piece of paper, and optimal dimensions are determined.
2. Optimal Tent Size (A4). A pyramidal shaped tent is cut from a square piece of canvas with maximal volume in two ways.
3. Optimal Foraging (A3). A study of seagulls dropping clams is examined for optimal foraging strategies.

Lab 4 (Help Page)

1. Length of Day (B3). A cosine function is used to approximate the length of the day over a year.
2. Fourier Fit to Population (D3). Data on hares gathered by the Hudson Bay company are fit with Fourier series.

Lab 5 (Help Page)

1. Optimal Trough (D1). A trough with a cross-section in the shape of an isosceles trapezoid is optimized for volume.
2. Tides (C2). Four cosine functions are fit to the October 2000 tide tables for San Diego and analyzed. Minima and maxima are explored.

Lab 6 (Help Page)

1. Atmospheric Pressure (F1). A simple model for atmospheric pressure is examined.

Math 122 Computer Lab Problems (cont)

2. Cell Study (F4). Compute the volume and surface area of different cells, then study their growth with a Malthusian growth law. Learn more about exponential growth testing a statement by Michael Crichton.
3. Radiocarbon Dating (E3). Radioactive decay of ^{14}C can be used to date ancient objects, using a simple linear differential equation.

Lab 7 (Help Page)

1. Malthusian and Logistic Growth Models (G1). The solutions of these models are explored with their slope fields using Maple.
2. Nonlinear Cell Growth (G4). A culture of cells is growing in a nonlinear and time-dependent manner. Solutions are found exactly and numerically.
3. Newton's Law of Cooling (G2). Newton's law of cooling is applied to a situation where a cat is killed by a car, and the time of death needs to be found.

Lab 8 (Help Page)

1. Euler's and Improved Euler's Methods (F2). Numerical solutions of two differential equations are studied.
2. Carbon Monoxide in a Room (I1). Machinery produces CO, which builds up in a room. Exposure levels are found by solving a differential equation exactly and numerically.

Lab 9 (Help Page)

1. Drug Absorption (G3). Two models for drug absorption are examined to show the difference between injected drugs and ones delivered using a polymer delivery system.
2. Growth of *E. coli* (H1). Two theories for the growth of the cytoplasm or mass of bacteria are compared.
3. Lead Exposure in Children (H2). Differential equations are used to find the level of lead in children during their early years.

Lab 10 (Help Page)

1. European Population Model (J1). A time-varying Malthusian growth model is used to help study the declining growth rates in several European countries.
2. Insect Population (I2). Polynomials and Fourier series are used to approximate a population survey. Definite integrals are used to find average populations.
3. Blood Flow in an Artery (J4). Poiseuille's law for flow of fluids is applied to small arteries. Integrals are used to derive relationships for the velocity of blood in arteries.

Student Learning

- Students appreciate math from biological context
- Relevant applications provide learning motivation
- Retention appears improved
- Multiple models – flexible approaches developed
- WeBWorK precision improves how students work
- Valuable computer skills learned

Resources

- Math 121/122 Website
- Text – Pearson Custom Publishing
- Lab Manual – New version under preparation
- WeBWorK
  - Link through course websites
  - Math 121
    login: GUEST1
    passwd: math121
  - Math 122
    login: GUEST2
    passwd: math122
- **Education Talks** – My research webpage