HW #9

7.10.1. Solve the initial value problem for the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\nabla^2 u$ inside a sphere of radius a subject to the boundary condition $u(a, \theta, \phi, t) = 0$ and the initial conditions

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(e) $u(\rho, \theta, \phi, 0) = F(\rho, \phi) \cos 3\theta$ and $\frac{\partial u}{\partial t}(\rho, \theta, \phi, 0) = 0$

7.10.2. Solve the initial value problem for the heat equation $\frac{\partial u}{\partial t} = k\nabla^2 u$ inside a sphere of radius a subject to the boundary condition $u(a, \theta, \phi, t) = 0$ and the initial conditions

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(c) $u(\rho, \theta, \phi, 0) = F(\rho, \phi) \cos\theta$

7.10.8. Differential equations related to Bessel's differential equation. Use this to show that

$$x^2\frac{d^2 f}{dx^2}+x(1-2a-2bx)\frac{df}{dx}+[a^2-p^2+(2a-1)bx+(d^2+b^2)x^2]f = 0 \quad (7.10.37)$$

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has solutions $x^a e^{bx} Z_p(dx)$, where $Z_p(x)$ satisfies Bessel's differential equation (7.7.25). By comparing (7.10.21) and (7.10.37), we have $a = -\frac{1}{2}, b = 0, \frac{1}{4} - p^2 = -n(n+1)$, and $d^2 = \lambda$. We find that $p = (n + \frac{1}{2})$.

7.10.9. Solve Laplace's equation inside a sphere $\rho < a$ subject to the following boundary conditions on the sphere:

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(a) $u(a, \theta, \phi) = F(\phi) \cos 4\theta$