HW #4

3.3.1. For the following functions, sketch $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$.

(c) $f(x) = \begin{cases} x & x < 0 \\ 1 + x & x > 0 \end{cases}$

3.3.14. (a) Consider a function $f(x)$ that is even around $x = L/2$. Show that the odd coefficients (n odd) of the Fourier cosine series of $f(x)$ on $0 \le x \le L$ are zero.

(b) Explain the result of part (a) by considering a Fourier cosine series of $f(x)$ on the interval $0 \le x \le L/2$.

3.4.6. There are some things wrong in the following demonstration. Find the mistakes and correct them.

In this exercise we attempt to obtain the Fourier cosine coefficients of e^x:

$$e^x = A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{n\pi x}{L}. \qquad (3.4.22)$$

Differentiating yields

$$e^x = -\sum_{n=1}^{\infty} \frac{n\pi}{L} A_n \sin \frac{n\pi x}{L},$$

the Fourier sine series of e^x. Differentiating again yields

$$e^x = -\sum_{n=1}^{\infty} \left(\frac{n\pi}{L}\right)^2 A_n \cos \frac{n\pi x}{L}. \qquad (3.4.23)$$

Since equations (3.4.22) and (3.4.23) give the Fourier cosine series of e^x, they must be identical. Thus,

$$\left.\begin{aligned} A_0 &= 0 \\ A_n &= 0 \end{aligned}\right\} \text{ (obviously wrong!).}$$

By correcting the mistakes, you should be able to obtain A_0 and A_n *without* using the typical technique, that is, $A_n = 2/L \int_0^L e^x \cos n\pi x/L \, dx$.

3.4.11. Consider the *nonhomogeneous* heat equation (with a steady heat source):

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} + g(x).$$

Solve this equation with the initial condition

$$u(x, 0) = f(x)$$

and the boundary conditions

$$u(0, t) = 0 \text{ and } u(L, t) = 0.$$

Assume that a continuous solution exists (with continuous derivatives). [*Hints:* Expand the solution as a Fourier sine series (i.e., use the method of eigenfunction expansion). Expand $g(x)$ as a Fourier sine series. Solve for the Fourier sine series of the solution. Justify all differentiations with respect to x.]