HW # 3

*2.4.1. Solve the heat equation $\partial u/\partial t = k\partial^2 u/\partial x^2$, $0 < x < L$, $t > 0$, subject to

$$\frac{\partial u}{\partial x}(0,t) = 0 \quad t > 0$$
$$\frac{\partial u}{\partial x}(L,t) = 0 \quad t > 0.$$

7 (a) $u(x,0) = \begin{cases} 0 & x < L/2 \\ 1 & x > L/2 \end{cases}$ 8 (c) $u(x,0) = -2\sin\frac{\pi x}{L}$

*2.4.2. Solve

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \quad \text{with} \quad \frac{\partial u}{\partial x}(0,t) = 0$$
$$u(L,t) = 0$$
$$u(x,0) = f(x).$$

For this problem you may assume that no solutions of the heat equation exponentially grow in time. You may also guess appropriate orthogonality conditions for the eigenfunctions.

2.5.1. Solve Laplace's equation inside a rectangle $0 \le x \le L$, $0 \le y \le H$, with the following boundary conditions:

10 (d) $u(0,y) = g(y)$, $u(L,y) = 0$, $\frac{\partial u}{\partial y}(x,0) = 0$, $u(x,H) = 0$

10 (g) $\frac{\partial u}{\partial x}(0,y) = 0$, $\frac{\partial u}{\partial x}(L,y) = 0$, $u(x,0) = \begin{cases} 0 & x > L/2 \\ 1 & x < L/2 \end{cases}$, $\frac{\partial u}{\partial y}(x,H) = 0$

2.5.2. Consider $u(x,y)$ satisfying Laplace's equation inside a rectangle ($0 < x < L$, $0 < y < H$) subject to the boundary conditions

$$\frac{\partial u}{\partial x}(0,y) = 0 \quad \frac{\partial u}{\partial y}(x,0) = 0$$
$$\frac{\partial u}{\partial x}(L,y) = 0 \quad \frac{\partial u}{\partial y}(x,H) = f(x).$$

15 2 *(a) *Without* solving this problem, briefly explain the physical condition under which there is a solution to this problem.

8+2 (b) Solve this problem by the method of separation of variables. Show that the method works only under the condition of part (a).

3 (c) The solution [part (b)] has an arbitrary constant. Determine it by consideration of the time-dependent heat equation (1.5.11) subject to the initial condition

$$u(x,y,0) = g(x,y).$$

2.5.6. Solve Laplace's equation inside a semicircle of radius a ($0 < r < a$, $0 < \theta < \pi$) subject to the boundary conditions

8 (b) the diameter is insulated and $u(a,\theta) = g(\theta)$

2.5.8. Solve Laplace's equation inside a circular annulus ($a < r < b$) subject to the boundary conditions

12 (b) $\frac{\partial u}{\partial r}(a,\theta) = 0$, $u(b,\theta) = g(\theta)$.

5 2.5.10. Using the maximum principles for Laplace's equation, prove that the solution of Poisson's equation, $\nabla^2 u = g(x)$, subject to $u = f(x)$ on the boundary, is unique.

2.5.15. Solve Laplace's equation inside a semi-infinite strip ($0 < x < \infty$, $0 < y < H$) subject to the boundary conditions

10 (b) $u(x,0) = 0$, $u(x,H) = 0$, $u(0,y) = f(y)$