1. (6pts) a. From the lecture notes, the concentration of pesticide satisfies the model equation:

$$\frac{dc}{dt} = \frac{f}{V}\left(20e^{-0.0005t} - c(t)\right),$$

where $f = 800$ and $V = 400,000$, so $\frac{f}{V} = 0.002$. Written in standard linear form, we have:

$$\frac{dc}{dt} + 0.002c(t) = 0.04e^{-0.0005t}.$$

The integrating factor becomes $\mu(t) = e^{\int 0.002dt} = e^{0.002t}$, so

$$\frac{d}{dt}\left(e^{0.002t}c(t)\right) = 0.04e^{0.0015t}.$$

Integrating gives;

$$e^{0.002t}c(t) = \frac{.04}{.0015}e^{0.0015t} + C \qquad \text{or} \qquad c(t) = \frac{80}{3}e^{-.0005t} + Ce^{-0.002t}.$$

With the initial condition $c(0) = 0$, we have $C = -\frac{80}{3}$, so

$$c(t) = \frac{80}{3}\left(e^{-.0005t} - e^{-0.002t}\right).$$

b. To find the time of the maximum concentration, solve $\frac{dc(t)}{dt} = 0$, which gives:

$$\frac{80}{3}\left(-0.0005e^{-.0005t} + 0.002e^{-0.002t}\right) = 0.$$

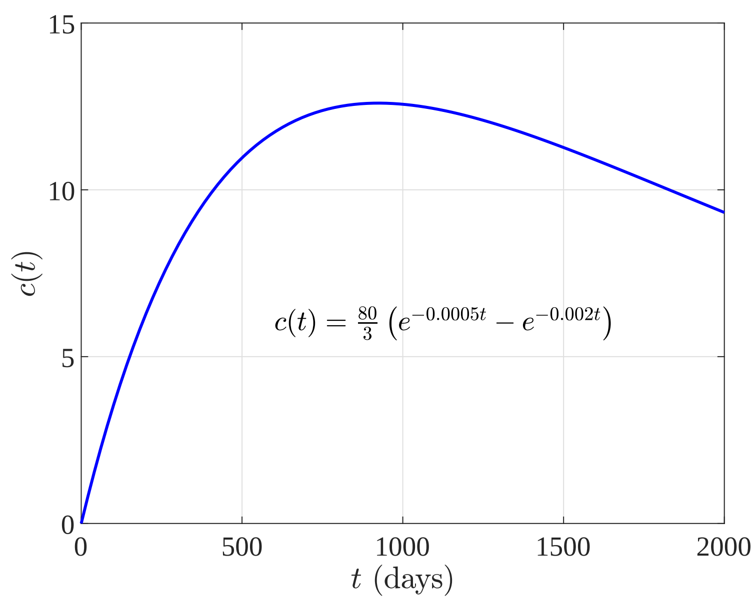
It follows that

$$0.0005e^{-.0005t} = 0.002e^{-0.002t} \qquad \text{or} \qquad e^{0.0015t} = 4.$$

Thus,

$$0.0015t = \ln(4) \qquad \text{or} \qquad t_{max} = \frac{2000\ln(4)}{3} \approx 924.196 \text{ da.}$$

The concentration at that time is $c(924.196) = 12.599\ \mu g/m^3$. The graph is shown below.



2. (5pts) a. The von Bertalanffy's growth equation for the blue shark is:

$$\frac{dL}{dt} = b(240 - L) \qquad \text{or} \qquad \frac{dL}{dt} + bL = 240b,$$

with $L(0) = 24$. The integrating factor is $\mu(t) = e^{\int b dt} = e^{bt}$, so

$$\frac{d}{dt}\left(e^{bt}L\right) = 240be^{bt}.$$

Integration yields

$$e^{bt}L = 240e^{bt} + C, \qquad \text{or} \qquad L(t) = 240 + Ce^{-bt}.$$

With $L(0) = 24$, it follows that $C = -216$, so

$$L(t) = 240 - 216e^{-bt}.$$

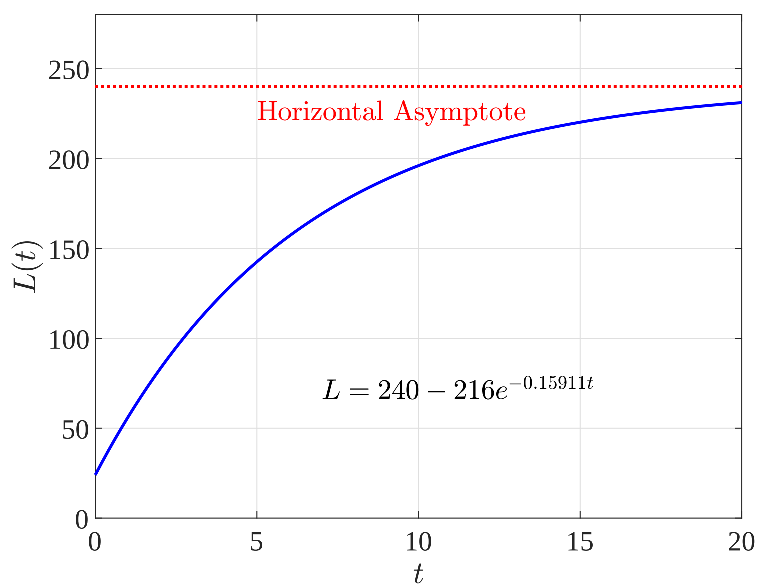
b. The blue shark satisfies $L(10) = 196$ cm, so $196 = 240 - 216e^{-10b}$. With algebra this gives:

$$e^{10b} = \frac{216}{44} \qquad \text{or} \qquad b = \frac{1}{10}\ln\left(\frac{216}{44}\right) \approx 0.15911.$$

It follows that

$$L(t) = 240 - 216e^{-0.15911t}.$$

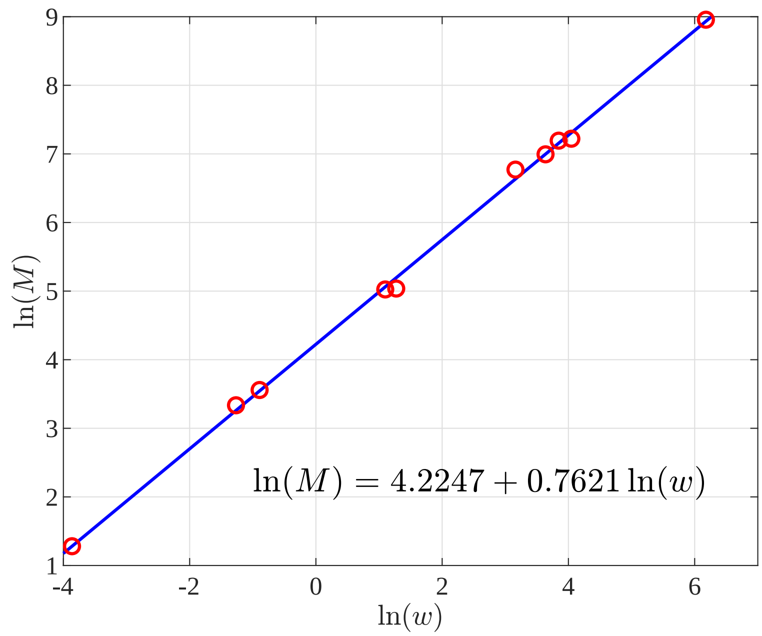
There is a horizontal asymptote at $L = 240$, and the graph is below.



3. (5pts) The best linear fit to the logarithms of the metabolic data satisfies:

$$\ln(M) = 0.7621\ln(w) + 4.2247,$$

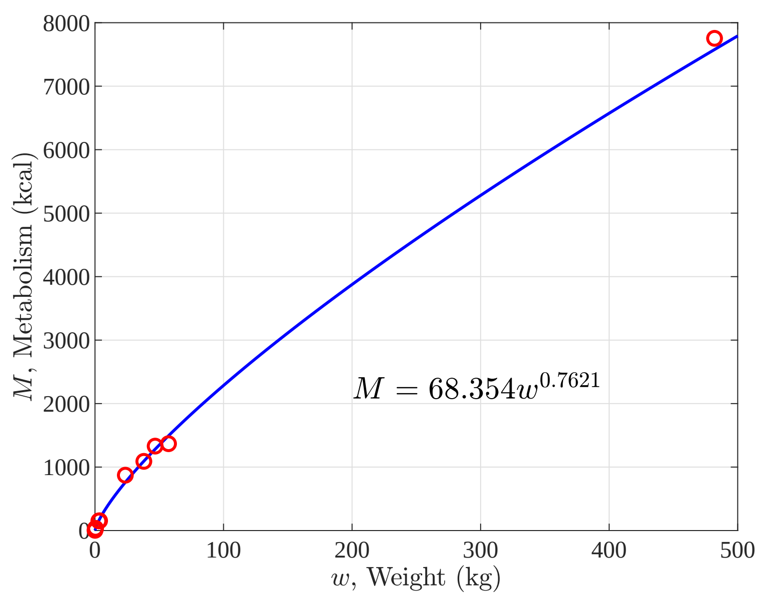
which is shown in the graph below:



This allometric model is given by:

$$M = 68.35\, w^{0.7621},$$

which is shown in the graph below:



Mammals use most of their energy generating heat, which is lost through the skin (surface area L^2). As weight is density times volume and the density is constant (mostly water), so weight is proportional to L^3. It follows that energy lost through the skin is proportional to $w^{2/3}$. However, mammals also use energy that is volumetric (proportional to w^1) through use of muscles, brain, digestion, and lungs. Thus, we expect the power in the allometric model to be greater than $2/3$ (and less than 1), weighted closer to $2/3$. These data give Kleiber's Law, which has $r = 0.75$ in agreement with the best fitting allometric model.