1. (4pts) The Malthusian growth model is:

$$\frac{dP}{dt} = rP, \quad \text{with} \quad P(0) = 75,$$

which has the solution

$$P(t) = 75e^{rt}.$$

Since $P(10) = 75e^{10r} = 120$, it follows that $e^{10r} = \frac{120}{75} = \frac{8}{5} = 1.6$. Thus, $10r = \ln(1.6)$ or $r = 0.1\ln(1.6) \approx 0.0470$. The doubling time, t_d satisfies

$$e^{rt_d} = 2 \qquad \text{or} \qquad t_d = \frac{\ln(2)}{r} \approx 14.75.$$

It follows that $P(50) = 75e^{50r} = 786.4$.

2. (4pts) The damped harmonic oscillator given by:

$$y'' + 4y' + 5y = 0,$$

is a linear homogeneous second order differential equation. We differentiate to see

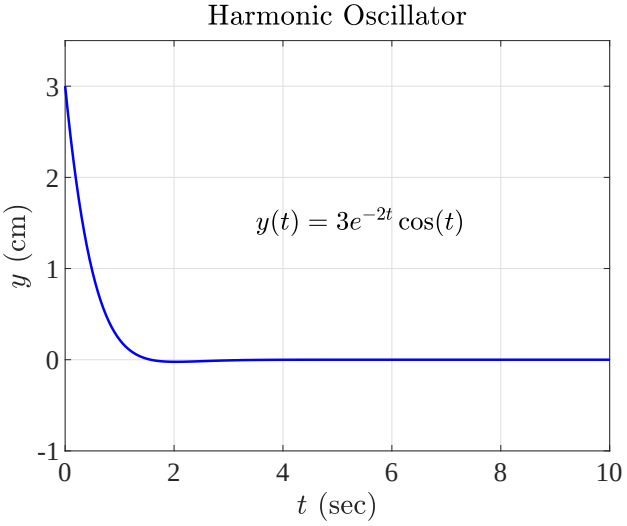
$$\begin{aligned} y(t) &= 3e^{-2t}\cos(t), \\ y'(t) &= 3e^{-2t}(-\sin(t) - 2\cos(t)), \\ y''(t) &= -3e^{-2t}\Big((\cos(t) - 2\sin(t)) - 2(\sin(t) + 2\cos(t))\Big) = e^{-2t}(9\cos(t) + 12\sin(t)). \end{aligned}$$

Substituting this into the damped harmonic oscillator gives:

$$e^{-2t}\Big((9\cos(t) + 12\sin(t)) + 4(-3\sin(t) - 6\cos(t)) + 5(3\cos(t))\Big) = 0,$$

which shows this is a solution to the ODE.

3. (4pts) The graph of the harmonic oscillator is below:



The MatLab program is below:

```
clear
figure(101)
clf
hold off
t = linspace(0,20,400);
y = 3*exp(-2*t).*cos(t);
plot(t,y,'b-','LineWidth',1.5);
grid;
xlim([0,10]);
ylim([-1,3.5]);
text(3.5,1.5,'$y(t) = 3e^{-2t}\cos(t)$','FontSize',16, ...
    'interpreter','latex');
fontlabs = 'Times New Roman';
xlabel('$t$ (sec)','FontSize',16,'FontName',fontlabs, ...
    'interpreter','latex');
ylabel('$y$ (cm)','FontSize',16,'FontName',fontlabs, ...
    'interpreter','latex');
mytitle = 'Harmonic Oscillator';
title(mytitle,'FontSize',16,'FontName',fontlabs);
set(gca,'FontSize',16);
print -depsc harmonic.eps
```

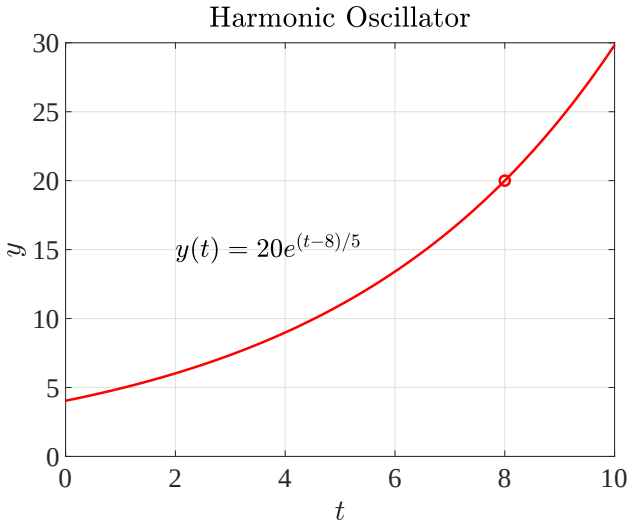
4. (4pts) The solution of the differential equation (ODE):

$$\frac{dy}{dt} = \frac{y}{5}, \quad \text{with} \quad y(8) = 20,$$

is given by:

$$y(t) = 20e^{(t-8)/5}.$$

This is a first order, linear, homogeneous differential equation. Below is a graph of the solution, where the circle denotes the initial condition.



The MatLab program is below:

```
clear
figure(101)
clf
hold off
t = linspace(0,10,200);
y = 20*exp((t-8)/5);
plot(t,y,'r-','LineWidth',1.5);
hold on
plot(8,20,'ro','LineWidth',1.5);
grid;
xlim([0,10]);
ylim([0,30]);
text(2,15,'$y(t) = 20e^{(t-8)/5}$','FontSize',16, ...
    'interpreter','latex');
fontlabs = 'Times New Roman';
xlabel('$t$','FontSize',16,'FontName',fontlabs, ...
    'interpreter','latex');
ylabel('$y$','FontSize',16,'FontName',fontlabs, ...
    'interpreter','latex');
mytitle = 'Harmonic Oscillator';
title(mytitle,'FontSize',16,'FontName',fontlabs);
set(gca,'FontSize',16);
print -depsc act1_4.eps
```