

This Lecture Activity is designed to have you actively work with the lecture notes presented in class and available on my website. This activity is meant to keep you engaged and current with the class, so there is a fairly rapid turn around (due by **Mon. Feb 28 by noon**). There are 3 problems that require written answers, which are entered into **Gradescope**.

Note: For full credit you must show intermediate steps in your calculations.

1. (4pts) Consider the initial value problem:

$$\frac{dy}{dt} = 3y - 20y^2e^{2t}, \quad \text{with } y(0) = 2.$$

Solve this initial value problem. (Slides Exact 19–26)

2. (4pts) Consider the initial value problem:

$$2y(4 - \sin(3t)) \frac{dy}{dt} = 3y^2 \cos(3t) - 8t^3, \quad \text{with } y(0) = 2$$

(Slides Exact 7–15)

3. (8pts) a. (**Give all constants to 4 significant figures.**) The population of the United Kingdom in 1960 was 52.4 million. The population increased to 56.3 million by 1980. Let $t = 0$ be associated with 1960 and t be measured in years. If the United Kingdom's population continued to grow according to the Malthusian growth law, then its population would satisfy the differential equation:

$$\frac{dP}{dt} = rP, \quad P(0) = 52.4.$$

With the information that $P(20) = 56.3$, solve this differential equation and find the growth constant r .

b. A modified Malthusian growth model with a declining linear growth fit the population growth of many countries much better. This model is given by:

$$\frac{dP}{dt} = (a - bt)P,$$

for some parameters a and b . Use the additional information that United Kingdom's population in 2000 is 59.5 million, so $P(40) = 59.5$. With all of the census data, solve the differential equation find the constants a and b . Find the predicted population in 2020. Also, find when this model predicts the population of the United Kingdom to peak and what is that predicted maximum population. (Slides Separable 26–34)