This assignment is your second Lecture Activity to have you actively work with the lecture notes presented in class and available on my website. This activity is meant to keep you engaged and current with the class, so there is a fairly rapid turn around (due by **Sat. Sep 4 by noon**). There are 4 problems that require written answers, which are entered into **Gradescope**.

Note: For full credit you must show intermediate steps in your calculations.

1. (4pts) a. We would like to determine whether a cup of tea cools more rapidly by adding cold milk right after brewing the tea or if you wait 5 min to add the milk. Begin with $\frac{4}{5}$ cup of boiling hot tea, $T(0) = 100°$C. Assume the tea cools according to Newton's law of cooling:

$$\frac{dT}{dt} = -k(T(t) - T_e) \qquad \text{with} \qquad T_e = 20°\text{C},$$

where k is the cooling constant based on the properties of the cup and water/milk to be calculated. Assume that after 2 min the tea has cooled to a temperature of $95°$C. Find k and the temperature after 5 min. Subsequently, add $\frac{1}{5}$ cup of cold milk, $5°$C. Stir in the cold milk, assuming that the temperature mixes perfectly in proportion to the volume of the two liquids. What is the temperature of the tea/milk mixture at 5 min? (Slides Intro 6-12)

b. (4pts) Now consider the case where again the cup of tea is cooling according to Newton's law of cooling with the equation above. However, the cold milk is added immediately and the mixture is allowed to cool for 5 min. Thus, assume that $\frac{4}{5}$ cup of boiling hot tea, $100°$C, is mixed with $\frac{1}{5}$ cup of cold milk, $5°$C. This mixture is assumed to cool with the same cooling constant k from Part a, so determine the temperature after 5 min and compare to your answer in Part a. Which strategy produces the hotter tea after 5 min?

2. (4pts) Consider the ODE given by:

$$\frac{dy}{dt} = 7 - 0.5y.$$

Solve this ODE. Find all equilibria. Determine the stability of all equilibria. Draw a phase portrait for this ODE. (Slides Qual Methods 21-28)

3. (4pts) Consider the ODE given by:

$$\frac{dy}{dt} = 0.5y^2 - 0.4y - 0.1y^3.$$

Find all equilibria. Determine the stability of all equilibria. Draw a phase portrait for this ODE. (Slides Qual Methods 21-28)