A tone (specific sound wave) enters the ear at a certain frequency, vibrating the tympanic membrane, which transfers the sound through small bones in the middle ear to the inner ear. The inner ear is filled with fluid and contains the cochlea, which can be thought of as a tube formed by a rolled up carpet. It is covered with about 15,000 *hair cells*, which have tapered projections that respond to a specific frequency of sound waves in the fluid. When hair cells die, they are not replaced. Typically in humans the high frequency hair cells are lost before the age of 20. The use of ear buds has exacerbated the loss of lower and mid-frequency range hair cells.

The ear uses a response similar to the resonance model in the SecondDE lecture notes (Slides 30-32), to determine what tone it is hearing.[1] A simplified model for the ear has the cilia of the hair cells along the cochlea acting like damped harmonic oscillators, each tuned to respond to a different frequency of incoming sound. Each cilium is associated with a nerve that detects motion and produces an electrical impulse response with sufficient stimulus. The sound wave behaves like the forcing function in the lecture problem.

a. Let the mass be one unit and assume that time,t, is in milliseconds. The displacement of the cilia, y, is given by the differential equation:

$$\frac{d^2y}{dt^2} + 2c\frac{dy}{dt} + k^2y = F_0\sin(at), \tag{1}$$

where $2c$ is the damping coefficient and k^2 is the *spring constant* for the cilia. The specific sound wave in the cochlea satisfies the forcing function with an amplitude of F_0 and frequency of a. Assume that the cilia are at rest initially, so the initial conditions are $y(0) = 0$ and $\dot{y}(0) = 0$. The physical properties of the fluid and the cilia constrain $k^2 > c^2$. Find the unique solution to this initial value problem (IVP). What is the asymptotic or limiting behavior for this problem?

b. Since the cilia are in the same fluid, assume that the damping coefficient always satisfies $c = 0.1$. If the test sound waves have the same intensity (amplitude), then we can let $F_0 = 5$. This leaves two parameters that we will vary, k^2 and a. Consider two cilia with $k^2 = 9$ for one and $k^2 = 12$ for the other. Each of these cilia are tested against six different frequency sound waves. The test sound waves have $a = 2.85$, 3.0, 3.15, 3.3, 3.45, and 3.6 (or $f = 454$, 477, 501, 525, 549, and 573 Hz, respectively, with $a = \frac{2\pi f}{1000}$).

Part a gives the unique solution to this problem for any parameters. We also want to explore solving this 2^{nd} order ODE using MatLab's *ode45* solver, which requires transforming the ODE into a 1^{st} order system of ODEs (Lecture SecondDE - Slide 6). Using either of these methods, you will create **6** graphs, finding $y(t)$ for $t \in [0, 50]$ at each of the **6** values of a with the two solutions from the two distinct $k^2 = 9$ and 12 values on each graph. For the simulations here, we want resolution in microseconds, so divide the interval $t \in [0, 50]$ into stepsizes of 0.001 (which in MatLab is $t =$ *[0:0.001:50]*, making a vector length 50,001). Write a brief description of what you observe in these graphs. In particular, note when the cilia have sustained oscillations exceeding an amplitude of 5 (more than 10 periods), giving the hair cells adequate stimulation to trigger a nerve cell to send a signal to the brain. For the cases where there is sufficient stimulation, determine the

[1] Tobias Reichenbach and A. J. Hudspeth, 'The physics of hearing: fluid mechanics and the active process of the inner ear,' https://arxiv.org/ftp/arxiv/papers/1408/1408.2085.pdf

actual solution at times $t = 30$ and $t = 50$ and find the percent error compared to the *ode45* solver at those times, so comment on how well the *ode45* solver tracks the actual solution. Also, find the maximum response for $t \in [0, 50]$, giving both $y(t_m)$ and t_m for each the two largest responses observed for $k^2 = 9$ and 12.

c. The particular solution consists of only a sine and cosine function, and the amplitude of this function is readily obtained (2DLinSysAppl - Slide 31), giving the maximum response for any of the parameters in (1). In this part of the problem, we want to observe the maximum response for distinct cilia with $k^2 = 8$, 9, 10, 11, 12, and 13 over a continuous range of sound frequencies, $a \in [2.7, 3.8]$. We want to display this result in a 3D graph of the amplitude vs the values of k^2 and a. The peaks will show the sound wave frequencies that most stimulate a particular hair cell. Compare the maximal response for $k^2 = 9$ and 12 with $a = 3.0$ and 3.45 to the numerical values in Part b.

There is a special **MatLab Help** page that is available through the hyperlink: Computer Activity 4 Help.