HW 2

1.5.21. Using (1.5.23) and (1.5.24), derive the Laplacian in an orthogonal curvilinear coordinate system:

$$\nabla^2 T = \frac{1}{h_u h_v h_w} \left[\frac{\partial}{\partial u} \left(\frac{h_v h_w}{h_u} \frac{\partial T}{\partial u} \right) + \frac{\partial}{\partial v} \left(\frac{h_u h_w}{h_v} \frac{\partial T}{\partial v} \right) + \frac{\partial}{\partial w} \left(\frac{h_u h_v}{h_w} \frac{\partial T}{\partial w} \right) \right].$$
$$(1.5.25)$$

⑤ 1.5.22. Using (1.5.25), derive the Laplacian for cylindrical coordinates.

④ 2.2.4. In this exercise we derive superposition principles for nonhomogeneous problems.

(a) Consider $L(u) = f$. If u_p is a particular solution, $L(u_p) = f$, and if u_1 and u_2 are homogeneous solutions, $L(u_i) = 0$, show that $u = u_p + c_1 u_1 + c_2 u_2$ is another particular solution.

(b) If $L(u) = f_1 + f_2$, where u_{pi} is a particular solution corresponding to f_i, what is a particular solution for $f_1 + f_2$?

⑫ 2.3.1. For the following partial differential equations, what ordinary differential equations are implied by the method of separation of variables?

$* (a) \quad \dfrac{\partial u}{\partial t} = \dfrac{k}{r} \dfrac{\partial}{\partial r} \left(r \dfrac{\partial u}{\partial r} \right)$

$(b) \quad \dfrac{\partial u}{\partial t} = k \dfrac{\partial^2 u}{\partial x^2} - v_0 \dfrac{\partial u}{\partial x}$

$* (c) \quad \dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

$(d) \quad \dfrac{\partial u}{\partial t} = \dfrac{k}{r^2} \dfrac{\partial}{\partial r} \left(r^2 \dfrac{\partial u}{\partial r} \right)$

$* (e) \quad \dfrac{\partial u}{\partial t} = k \dfrac{\partial^4 u}{\partial x^4}.$

$* (f) \quad \dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$

⑮ *2.3.8. Consider

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2} - \alpha u.$$

This corresponds to a one-dimensional rod either with heat loss through the lateral sides with outside temperature $0°$ ($\alpha > 0$, see Exercise 1.2.4) or with insulated lateral sides with a heat sink proportional to the temperature. Suppose that the boundary conditions are

$$u(0, t) = 0 \quad \text{and} \quad u(L, t) = 0.$$

(a) What are the possible equilibrium temperature distributions if $\alpha > 0$?

(b) Solve the time-dependent problem $[u(x, 0) = f(x)]$ if $\alpha > 0$. Analyze the temperature for large time ($t \to \infty$) and compare to part (a).