HW 11

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10 10.3.1. Show that the Fourier transform is a linear operator; that is, show that

5 (a) $\mathcal{F}[c_1 f(x) + c_2 g(x)] = c_1 F(\omega) + c_2 G(\omega)$

5 (b) $\mathcal{F}[f(x)g(x)] \neq F(\omega)G(\omega)$

5 10.3.5. If $F(\omega)$ is the Fourier transform of $f(x)$, show that the inverse Fourier transform of $e^{i\omega\beta}F(\omega)$ is $f(x-\beta)$. This result is known as the **shift theorem** for Fourier transforms.

7 *10.3.6. If

$$f(x) = \begin{cases} 0 & |x| > a \\ 1 & |x| < a, \end{cases}$$

determine the Fourier transform of $f(x)$. [The answer is given in the table of Fourier transforms in Section 10.4.4.]

8 10.4.1. Using Green's formula, show that

$$\mathcal{F}\left[\frac{d^2 f}{dx^2}\right] = -\omega^2 F(\omega) + \frac{e^{i\omega x}}{2\pi}\left(\frac{df}{dx} - i\omega f\right)\Big|_{-\infty}^{\infty}.$$

20 10.4.3. *(a) Solve the diffusion equation with **convection**:

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$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + c\frac{\partial u}{\partial x} \qquad -\infty < x < \infty$$

$$u(x,0) = f(x).$$

[*Hint*: Use the convolution theorem and the shift theorem (see Exercise 10.4.5).]

10 (b) Consider the initial condition to be $\delta(x)$. Sketch the corresponding $u(x,t)$ for various values of $t > 0$. Comment on the significance of the convection term $c\,\partial u/\partial x$.