HW #10

8.2.2. Consider the heat equation with time-dependent sources and boundary conditions:

7pts

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + Q(x,t)$$
$$u(x,0) = f(x).$$

Reduce the problem to one with homogeneous boundary conditions if

(b) $u(0,t) = A(t)$ and $\frac{\partial u}{\partial x}(L,t) = B(t)$

8.2.5. Solve the initial value problem for a two-dimensional heat equation inside a circle (of radius a) with time-independent boundary conditions:

15pts

$$\frac{\partial u}{\partial t} = k\nabla^2 u$$
$$u(a,\theta,t) = g(\theta)$$
$$u(r,\theta,0) = f(r,\theta).$$

8.3.1. Solve the initial value problem for the heat equation with time-dependent sources

10pts

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + Q(x,t)$$
$$u(x,0) = f(x)$$

subject to the following boundary conditions:

(a) $u(0,t) = 0,$ $\frac{\partial u}{\partial x}(L,t) = 0$

8.4.2. Use the method of eigenfunction expansions to solve, without reducing to homogeneous boundary conditions:

15pts

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2}$$

$$u(x,0) = f(x) \qquad \left.\begin{aligned} u(0,t) &= A \\ u(L,t) &= B \end{aligned}\right\} \text{constants.}$$

8.4.3. Consider

15pts

$$c(x)\rho(x)\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left[K_0(x)\frac{\partial u}{\partial x}\right] + q(x)u + f(x,t)$$

$$u(x,0) = g(x) \qquad \begin{aligned} u(0,t) &= \alpha(t) \\ u(L,t) &= \beta(t). \end{aligned}$$

Assume that the eigenfunctions $\phi_n(x)$ of the related homogeneous problem are known.

(a) Solve without reducing to a problem with homogeneous boundary conditions.

(b) Solve by first reducing to a problem with homogeneous boundary conditions.

9.2.1. Consider

20pts

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} + Q(x,t)$$
$$u(x,0) = g(x).$$

In all cases obtain formulas similar to (9.2.20) by introducing a Green's function.

(c) Solve using any method if

$$\frac{\partial u}{\partial x}(0,t) = 0 \qquad \text{and} \qquad \frac{\partial u}{\partial x}(L,t) = 0.$$

*(d) Use Green's formula instead of term-by-term differentiation if

$$\frac{\partial u}{\partial x}(0,t) = A(t) \qquad \text{and} \qquad \frac{\partial u}{\partial x}(L,t) = B(t).$$