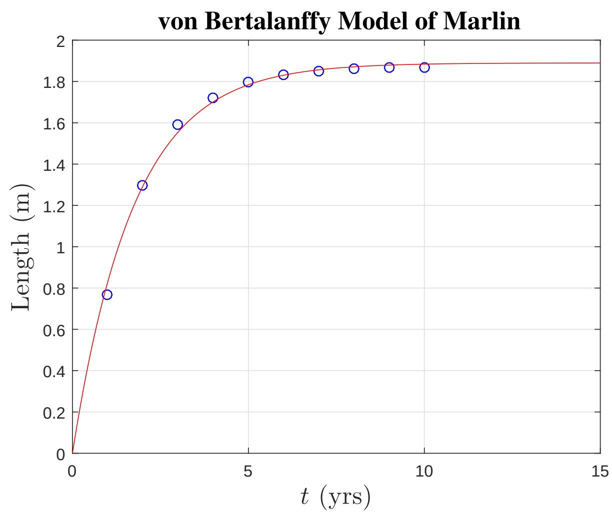
Homework Solutions – Due Fri. 02/15/2019

18. b. (4pts) (Numbers and species will vary with different versions from WeBWorK, but all solutions are similar in shape.) The graph of the best fitting model using the von Bertalanffy model for the Striped Marlin with the data set is seen below, fitting the model

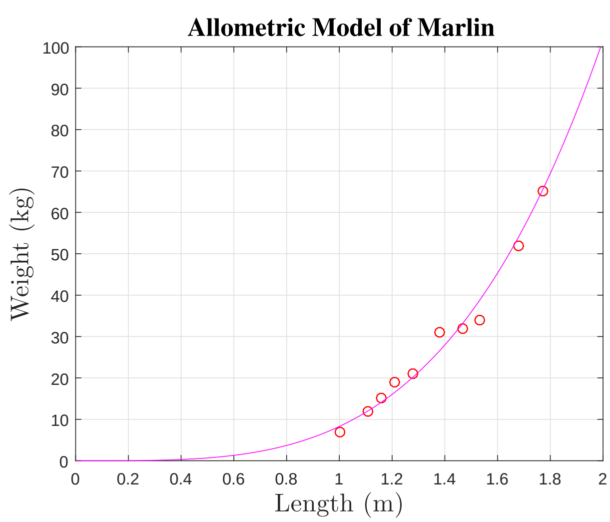
$$L(t) = 1.8898\left(1 - e^{-0.5764t}\right).$$



The graph shows that the rate of growth of the fish is very fast in the early years and then slows down, approaching an asymptote, as the fish ages. The model quite clearly matches the data very well. The graph shows that the maximum length the fish is the asymptotic limit, which is about 1.89 m.

d. (4pts)The graph of the best fitting allometric model for the Striped Marlin with the data set is seen below, using the model:

$$W(L) = 8.2861L^{3.6166}.$$



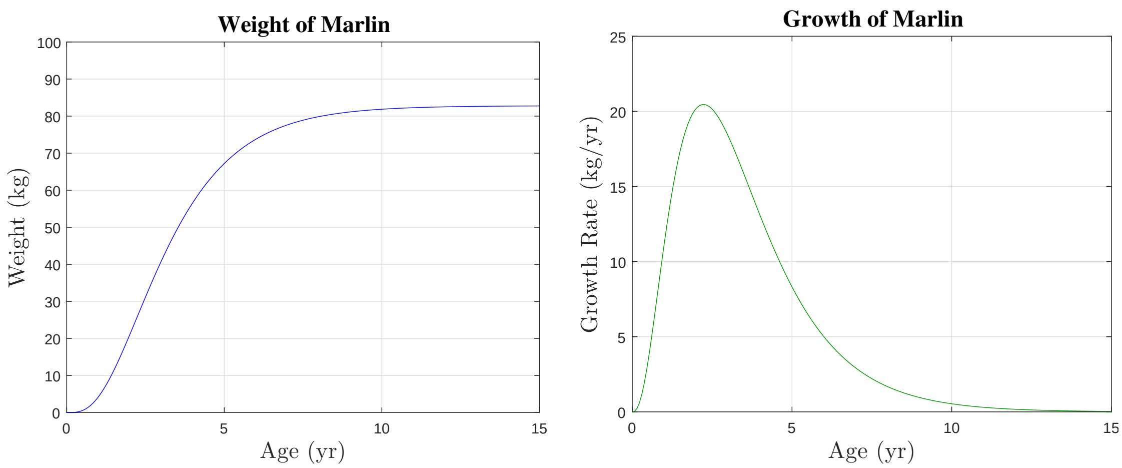
The graph and the model show that the allometric model roughly follows a cubic relationship between the length and the weight, which is expected based on dimensional analysis (between length and volume). The model quite clearly matches the data very well with a little more variation than seen in the previous graph. With the asymptotic limit of the length of the Striped Marlin from Part a, this model would indicate that the Striped Marlin has a limit in weight of approximately 82.8 kg.

f. (6pts) The functions above are combined in a composite function to give $W(t)$ for the Striped Marlin,

$$W(t) = 82.7983\left(1 - e^{-0.5764t}\right)^{3.6166}.$$

The growth function satisfies the derivative of $W(t)$ with

$$W'(t) = 172.6020184\left(1 - e^{-0.5764\,t}\right)^{2.6166} e^{-0.5764\,t}.$$



$W(t)$ $W'(t)$

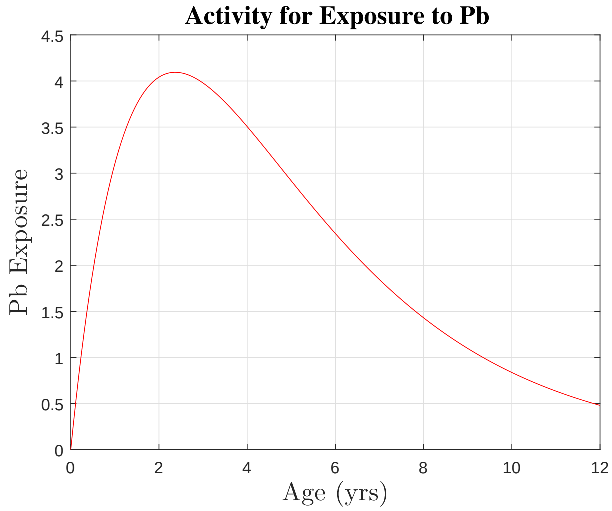
The graph of $W(t)$ shows the increase in weight as the fish ages with the increase accelerating for the first 2-3 years then slowing down as it approaches a maximum weight for large time. The point of inflection for the first graph matches the maximum of the growth curve on the right. The growth curve shows the increasing growth rate until approximately the age of 2.25 before growth slows to almost zero for older Marlin. This maximum growth shows the 2 year old Marlin putting on almost 20 kg/yr.

This assignment shows that the von Bertalanffy model, a linear ODE, reasonably fits measured data on Marlin. An allometric model to match data connecting length and weight performs quite well. This allows a relatively simple model (composite) connecting the increase in weight of Marlin as it ages, which can be important on determining sexual maturity and reproductive capability. The primary strength of the models are their simplicity and ability to match data. The primary weakness is that the models are based on limited data and fail to give much detail to explain why the specific values are obtained.

19. b. (4pts) (Details will vary with different versions from WeBWorK, but all solutions are similar in shape.) The linear ODE is readily solved to give:

$$A(t) = \frac{b}{q-k}\left(e^{-kt} - e^{-qt}\right).$$

A graph of this activity level leading to exposure to lead is given below.

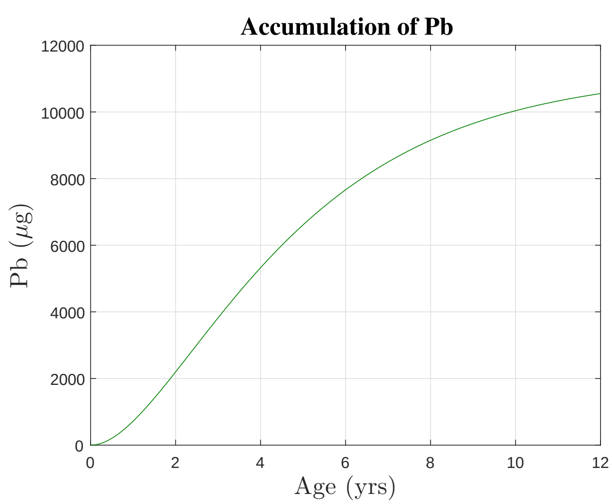


This model describes the amount of lead exposure a child may have based on their age (t in years). The model reasonably describes the lead exposure of a typical child, since as a child grows they go from being immobile (baby) to crawling around on dirty floors (infant), which may have lead, and chewing on things, which may also have lead. At a certain age around two, children stand and walk. Thus, they move away from the sources of lead contamination (ground dirt and paint chipping), and they decrease their hand in the mouth activities. This graph shows that cycle pretty well. (Credit can be given for contradicting this model if it is reasonably backed up with good information about child behavior.)

d. (3pts) The lead (Pb) accumulation is found from a time-varying ODE, which has the solution:

$$P(t) = \frac{Kb}{q-k}\left(\frac{1-e^{-kt}}{k} - \frac{1-e^{-qt}}{q}\right).$$

A graph of this lead accumulation is given below.

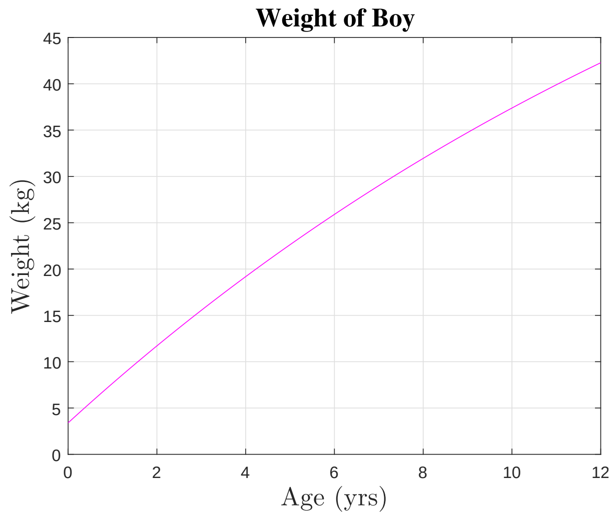


The model describes the accumulation of lead in a child. Since lead does not filter out of our bodies and simply accumulates, the graph shows in that the amount of lead only increases with age. Because activity at young ages gives the highest exposure to lead, the graph shows more accumulation of Pb at younger ages than at older ages.

e. (3pts) With the linear ODE describing the weight of the boy, the solution is readily found:

$$W(t) = W_{max} - (W_{max} - W_0)e^{-rt}.$$

A graph of this weight of a boy is given below.

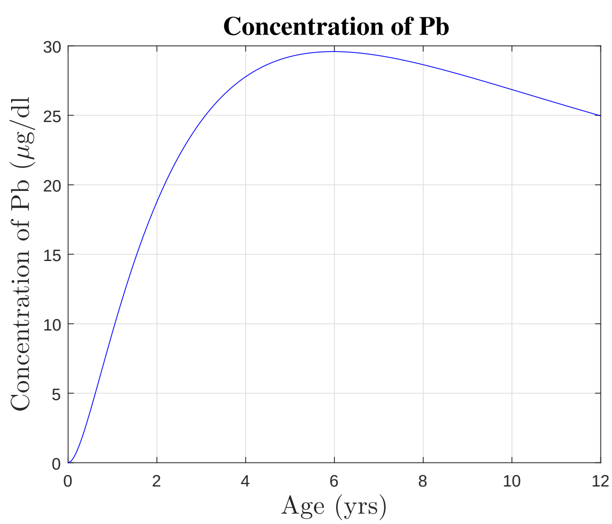


This model describes the weight of a child as he ages. The model produces a maximum weight of the person reaching W_{max}. Since this model is based on the von Bertalanffy equation for fish, we expect that it is only a very crude approximation for weight of a boy. A better model might match the weight model (cubic) done for the fish above and certainly fitting a model to actual data would improve modeling efforts.

g. (4pts) The concentration of lead is readily found from the formula:

$$c(t) = \frac{0.1P(t)}{W(t)}.$$

A graph of this concentration of Pb in a boy as he ages is given below.



This model describes the concentration of lead in a child over time, which based on the previous models appears to do a good job. The graph shows that the lead concentration in the body keeps rising to a peak concentration around 5 or 6 years of age. Later in development the lead intake increases more slowly than the weight gain, which decreases the concentration of lead. Thus, the toxicity actually declines after the peak. Still this particular case achieves concentrations approaching 30 μg/dl, which is far above acceptable levels. (Actually, no level of Pb in the blood is acceptable as it disrupts many bodily functions, especially neural development.) At the level in this particular case, the boy would likely have neurological damage and some hormonal problems. This child would probably need a chelating treatment to remove some of the lead and prevent slowed mental development.