Spring 2018 Math 541

Homework – Splines Due Mon. 4/16/18

Be sure to include all MatLab programs used to obtain answers.

1. a. Our program `cubic_splinenat.m` takes an input of data and outputs a matrix of coefficients, the interval for those coefficients, and a graphic display of the data with the best fitting cubic spline with natural boundary conditions. Modify this code to only output the matrix of coefficients. Take the function:

$$f(x) = 5e^{0.2x}, \qquad x \in [0, 4].$$

For your data evaluate $f(x)$ at the integers, $x = [0, 1, 2, 3, 4]$ and $y = [f(0), f(1), f(2), f(3), f(4)]$ to create your best spline fit. Give the best cubic spline $S_3(x)$ for $x \in [2, 3]$ based on your program. Since this is a cubic polynomial, then it is easy to integrate. Find

$$\int_2^3 S_3(x)dx, \qquad \text{and compare it to} \qquad \int_2^3 f(x)dx.$$

(Give the integral values and absolute error.)

b. In this part of the problem, we generalize the results from Part a to create a **quadrature program** based on cubic splines. In other words, we want to use piecewise cubic spline interpolation to build approximations to

$$\int_a^b f(x)dx.$$

Your code should take a, b, N, and f as arguments. It should divide your interval $[a, b]$ into N even steps, creating a vector of x-values, $x = [x_0, ..., x_N]$ with $x_0 = a$ and $x_N = b$. Evaluate the function at these points to get your y-values, $y = [f(x_0), ..., (x_N)]$. These x and y values are inserted into your cubic spline program to produce the coefficients on each subinterval, then the cubic polynomials are readily integrated on each subinterval. The total over all subintervals gives you the new **spline quadrature** for approximating our integral above. Use $f(x) = 5e^{0.2x}, x \in [0, 4]$ with $N = 4$ as a test of your program and show your results.

c. Finally, we want to determine the efficiency and convergence of this quadrature technique. Start with $N = 2$ ($h = 2$) and double N 5 times. Find the **absolute error** for each value of N and plot the log-log plot of the absolute error versus the stepsize h. Do the same with the Composite Trapezoid rule to compare routines. How do this spline quadrature and the Composite Trapezoid method compare in terms of accuracy and speed?

2. Below is a table of the U. S. population (in millions) from 1900 to 2000 (according to the U. S. Census Bureau).

Year	Population	Year	Population
1900	62.948	1960	150.697
1910	76.212	1970	179.323
1920	92.228	1980	203.302
1930	106.022	1990	226.546
1940	122.775	2000	248.710
1950	132.165		

a. Use the programs from class `polyinterp.m` and `cubic_splinenat.m` to graph (on a single plot) these data, showing the 10^{th} order polynomial and natural cubic spline, respectively, fitting the data above. Note which graph looks better and give a short discussion of why that technique is the superior method to fit these data.

b. In the class notes, we demonstrated how the Vandermonde matrix, V, can be used to obtain the coefficients of the interpolating polynomial. Give these 11 coefficients. Also, give the condition number (1-norm) for V. Find the largest and smallest of each of the coefficients of the cubic spline for this example and compare them to the ones found using V.

c. The large values for the years makes this problem quite ill-conditioned. (See Math 543 for more details in the future.) Change your data so that 1900 becomes $t = 0$. (Thus, 1950 is $t = 50$.) Repeat the steps in Parts a and b, and note any changes that occur. Would you predict that this change of variables improves the model fitting the data?