

1. a. In the lecture notes, we examined the ODE

$$y'' - y = 0.$$

Show that  $y_1(t) = e^t$  and  $y_2(t) = e^{-t}$  are solutions to the differential equation. In addition, show that this pair form a linearly independent set.

b. Also, show that  $y_1(t) = \sinh(t)$  and  $y_2(t) = \sinh(1 - t)$  are solutions to the differential equation. In addition, show that this pair form another linearly independent set.

2. Consider the following second order linear homogeneous differential equation:

$$y'' - 2ay' + (a^2 + b^2)y = 0.$$

a. Find the general solution to this ordinary differential equation (ODE).

b. Find the unique solution to the initial value problem (IVP), where the initial conditions for the ODE are:

$$y(0) = y_0 \quad \text{and} \quad y'(0) = z_0.$$

c. Now consider the ODE with boundary conditions:

$$y(0) = A \quad \text{and} \quad y(x_0) = B.$$

Give conditions on the parameters,  $A$ ,  $B$ , and  $x_0 > 0$ , such that this boundary value problem (BVP) has:

(i) A unique solution.      (ii) No solution.      (iii) Infinitely many solutions.

When this BVP has a unique solution, give the solution to the BVP.