April 14, 2017 Math 124a Name Key

1. (5pts) Evaluate the following integral:

$$-9x^{1/2}$$

$$\int \left(16\sin(4x) - 9\sqrt{x} - \frac{1}{3x}\right)\, dx.$$

Answer = $-4\cos(4x) - 6x^{3/2} - \frac{1}{3}\ln|x| + C$

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2. (7pts) a. White lead (^{210}Pb) is a pigment found in oil paints and can be used to detect art forgeries. In the absence of radium (^{226}Ra), (^{210}Pb) undergoes standard radioactive decay with a half-life of 22.2 yr. Suppose a particular paint pigment satisfies the differential equation when fresh:

$$\frac{dP}{dt} = -kP, \qquad P(0) = 8.5 \text{ dpm}.$$

Use the half-life of ^{210}Pb to find the value of k. A specific painting that contains this particular pigment is found to have reading of 1.2 dpm (disintegrations per minute). Determine the age of this painting using the current reading of 1.2 dpm.

$P(t) = 8.5e^{-kt} = 1.2 \quad e^{kt} = \frac{8.5}{1.2} \quad t = \frac{1}{k}\ln\left(\frac{8.5}{1.2}\right) \qquad k = \frac{\ln(2)}{22.2} = 0.031223$

$k =$ 0.031223 yr^{-1} Age of painting = 62.702 yr

b. The actual forgery is detected because the ^{226}Ra is removed in the manufacture of the pigment, leaving a higher concentration of ^{210}Pb. When impurities of ^{226}Ra remain (which has a very long half-life), the differential equation for radioactive decay is modified to

$$\frac{dP}{dt} = -kP + r, \qquad P(0) = 8.5 \text{ dpm}.$$

where $r = 0.005$ is source input from the remaining ^{226}Ra and k is from Part a. Solve this differential equation.

$\frac{dP}{dt} = -0.031223P + 0.005 = -0.031223(P - 0.16014)$

$z(t) = P(t) - 0.16014$

$P(t) =$ $8.3399e^{-0.031223t} + 0.1601$

$\frac{dz}{dt} = -kz, \quad z(0) = 8.5 - 0.16014$

$z(t) = 8.3399e^{-kt}$

3. (8pts) An elderly patient is running a fever of 39.3°C one night when her caregiver leaves. At 7 AM the next morning, the patient is found to have died from the fever. Her body temperature is taken immediately (at 7 AM) and found to be 32.4°C. Two hours later (at 9 AM), her body temperature is found to be 30.6°C. The room that she was staying was maintained at a constant temperature of 25°C. Assume that her body is cooled according Newton's Law of cooling,

$$\frac{dH}{dt} = -k_a(H - T_e),$$

where $H(t)$ is the temperature of the body, T_e is the room temperature, and k_a is the Newton's law cooling constant. Let $t = 0$ correspond to 7 AM, so $H(0) = 32.4$. Solve this differential equation, and use the information at 9 AM ($t = 2$) to find the value of k_a. Estimate the time of death assuming that the patient's temperature was 39.3°C at the time of death.

$\frac{dH}{dt} = -k_a(H-25)$, $\quad z(t) = H(t) - 25$, $\quad \frac{dz}{dt} = -k_a z$, $\quad z(0) = 32.4 - 25 = 7.4$

$z(t) = 7.4e^{-k_a t} = H(t) - 25$, $\quad H(t) = 25 + 7.4e^{-k_a t}$

$H(2) = 30.6 = 25 + 7.4e^{-2k_a} \Rightarrow 5.6 = 7.4e^{-2k_a} \Rightarrow e^{2k_a} = \frac{7.4}{5.6}$

$k_a = \frac{1}{2}\ln\left(\frac{7.4}{5.6}\right)$:

$k_a =$ 0.13936 $\qquad H(t) =$ $25 + 7.4e^{-0.13936t}$

Time of Death = 2:16.36 AM

$39.3 = 25 + 7.4e^{-0.13936t}$

$14.3 = 7.4e^{-0.13936t}$

$e^{0.13936t} = \ln\left(\frac{7.4}{14.3}\right)$

$t = \frac{1}{0.13936}\ln\left(\frac{7.4}{14.3}\right) = -4.72729$