April 7, 2017 Math 124b Name *Key*

1. (4pts) Technetium-99m (^{99}Tc) is an important isotope used in medical imaging, which lately is in short supply. Suppose that you begin with 21.0 μg, then the amount of ^{99}Tc satisfies the equation:

$$\frac{dR}{dt} = -kR \qquad R(0) = 21.0.$$

Assume that after 4 hours only 13.4 μg remain. Find the value of k and give the general solution to this differential equation. Also, determine the amount remaining after 30 hr.

$R(t) = 21\,e^{-kt}$ $\quad 13.4 = 21 e^{-4k} \Rightarrow e^{4k} = \frac{21}{13.4}$ $\quad k = \frac{1}{4} \ln\left(\frac{21}{13.4}\right)$

$k =$ 0.11232 $\qquad R(t) =$ $21\,e^{-0.11232t}$

$R(30) =$ 0.72254

2. (16pts) a. A study on the American Cockroach, *Periplaneta americana*, with a fix amount of food gives a best fitting logistic growth model of the form

$$P_{n+1} = F(P_n) = 2.28\,P_n - 0.0061\,P_n^2, \qquad P_0 = 5,$$

where n is in weeks. Find all equilibria and compute the derivative of $F(P)$. Determine the stability of the equilibria. Justify your stability argument by evaluating the derivative of the updating function. Credit for stability and behavior will be given only if information above the circled quantities is correct.

$P_e = 2.28P_e - 0.0061\,P_e^2$, so $P_e = 0$ or $1 = 2.28 - 0.0061\,P_e$

$\Rightarrow P_e = \frac{1.28}{0.0061}$

$F'(P) =$ $2.28 - 0.0122\,P$

$P_{1e} =$ 0 $\qquad F'(P_{1e}) =$ 2.28

Stable or Unstable $\qquad$ Monotonic or Oscillatory

$P_{2e} =$ 209.84 $\qquad F'(P_{2e}) =$ -0.28

Stable or Unstable $\qquad$ Monotonic or Oscillatory

b. An alternate model that is popular in ecology is Ricker's model. The best fitting version of this model for the data is given by:

$$P_{n+1} = R(P_n) = 2.94\, P_n e^{-P_n/330}, \qquad P_0 = 8.$$

Find $R'(P)$, then determine the maximum of this function (both P and $R(P)$ values).

$$R'(P) = 2.94\left(-\frac{P}{330} e^{-P/330} + e^{-P/330}\right)$$

$R'(P) =$ $2.94\, e^{-P/330}\left(1 - \frac{P}{330}\right)$

P-intercept 0 R-intercept 0 Horizontal Asymptote $R =$ 0

$P_{max} =$ 330 $R(P_{max}) =$ 356.92

c. Find the positive (non-extinction) equilibrium. Determine the stability of this equilibrium. Justify your stability argument by evaluating the derivative of the updating function. Credit for stability and behavior will be given only if information above the circled quantity is correct.

$P_e = 2.94\, P_e\, e^{-P_e/330}$, so $P_e = 0$ or $1 = 2.94\, e^{-P_e/330}$

$e^{P_e/330} = 2.94$

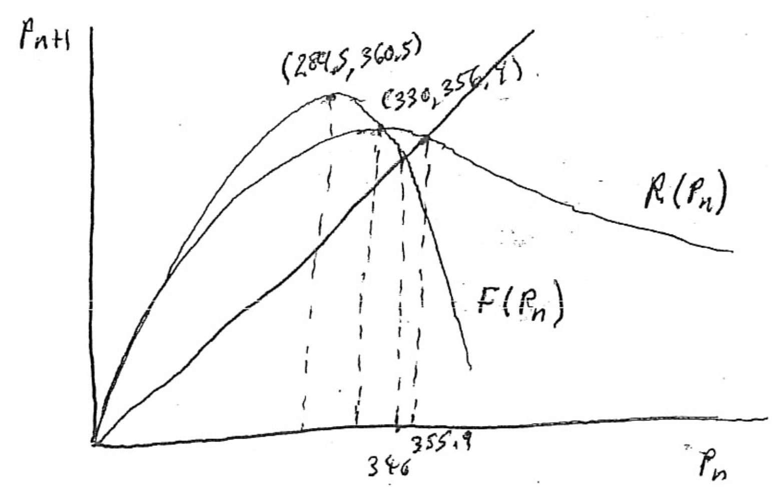
$P_e = 330 \ln(2.94)$

$P_e =$ 355.88 $R'(P_e) =$ -0.07841

Stable or Unstable Monotonic or Oscillatory

d. Sketch a graph of $F(P)$, $R(P)$, and the identity map. Show clearly the maxima and equilibria.

GRAPH:



$$F'(P) = 0 \Rightarrow p = \frac{2.49}{0.0086} = 289.53$$