February 10, 2017 Math 124a Name Key

1. (7pts) a. A study of birds in flight showed an allometric relationship between the weight of the bird and the speed of its flight. The Common Swift (*Apus apus*) had a weight (M) of 0.038 kg and flew at a velocity (U) of 10.6 m/sec. The Barnacle Goose (*Branta leucopsis*) had a weight (M) of 1.705 kg and flew at a velocity (U) of 17.0 m/sec. (Give all answers to **4 significant figures**.) Use an allometric model of the form:

$$U = cM^a. \qquad \ln(U) = \ln(c) + a\ln(M)$$

Find the constants c and a. (Give all answers to **4 significant figures**.)

M	$\ln(M)$	U	$\ln(U)$
0.038	-3.27017	10.6	2.360854
1.705	0.533565	17.0	2.833213

$$a = \frac{\ln(U_2) - \ln(U_1)}{\ln(M_2) - \ln(M_1)} = \frac{2.833213 - 2.360854}{0.533565 + 3.27017}$$

$$\ln(c) = \ln(U_1) - a\ln(M_1) = 2.36085 + a(3.27017) = 2.76695$$

$c =$ 15.9101 $a =$ 0.124183

b. The Mistle Thrush (*Turdus viscivorus*) weighs 0.114 kg, so use this model to predict the velocity, U, at which the Mistle Thrush flies. Also, The Pomerine Jaeger (*Stercorarius pomarinus*) was clocked with a speed of 15.2 m/sec, so use this model to predict the mass, M, of the Pomerine Jaeger.

$U = 15.9101\, M^{0.124183}$ $\quad 15.9101\,(0.114)^{0.124183}$

$15.2 = 15.9101\, M^{0.124183}$

$$M = \left(\frac{15.2}{15.9101}\right)^{1/0.124183}$$

Thrush: $U =$ 12.1494 m/sec

Jaeger: $M =$ 0.69235 kg

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2. (4pts) For the function,

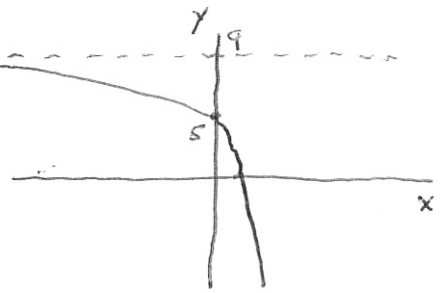
$$y = 9 - 4e^{x/2}$$

Find the x and y-intercepts and the horizontal asymptote along with its direction. Sketch the graph.

$9 = 4e^{x/2} \Rightarrow e^{x/2} = 9/4$

$x = 2\ln(9/4)$

Graph:



x-intercept 1.62186 y-intercept 5

Horizontal Asymptote: $y = 9$ **Circle One:** **To the Right** or **To the Left**

$T = \frac{2\pi}{\pi/7} = 14$

3. (9pts) a. Consider the trigonometric function:

$$y(t) = 6 - 9\cos\left(\frac{\pi}{7}(t-4)\right), \qquad t \in [0, 15].$$

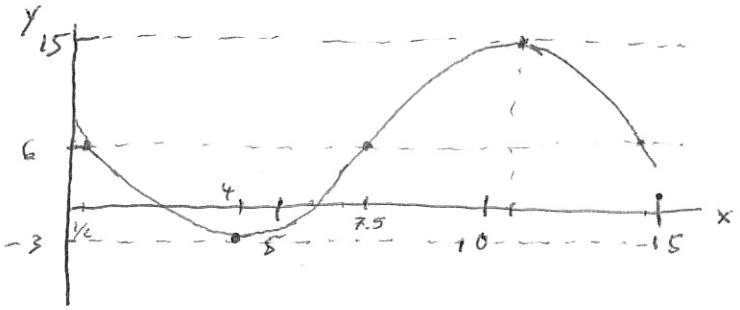
Find the period, amplitude, phase shift, and vertical shift of this function. Give the t and y values for all absolute maxima $(t_{max}, y(t_{max}))$ and absolute minima $(t_{min}, y(t_{min}))$ in the specified interval. (Note that there could be more than one maximum or minimum.) Sketch the graph of this function.

Period = 14 Amplitude = 9 $\frac{14}{4} = 3.5$

Phase Shift = 4 Vertical Shift = 6

$(t_{max}, y(t_{max}))$ = (11, 15) $(t_{min}, y(t_{min}))$ = (4, -3)

GRAPH:



b. Create an equivalent model in the form:

$$y(t) = 6 + 9\cos\left(\frac{\pi}{7}(t-\phi)\right),$$

with $\phi \in [0, T)$, where T is the period of the function.

Max at $t = 11$

$\Rightarrow \phi = 11$

$\phi =$ 11

c. Create an equivalent model in the form:

$$y(t) = 6 + 9\sin\left(\frac{\pi}{7}(t-\psi)\right),$$

with $\psi \in [0, T)$, where T is the period of the function.

Max at $t = 11$

$\Rightarrow \frac{\pi}{7}(11 - \psi) = \frac{\pi}{2}$

$\psi = 11 - \frac{7}{2} = 7.5$

$\psi =$ 7.5