1. a. For the logistic growth model, the best values of the parameters that fit the data are $r = 1.558$ and $m = 0.001654$, which gives the best fit model

$$P_{n+1} = 1.558P_n - 0.001654P_n^2.$$

The least sum of square errors between the data and this model is 4620.92.

For the Beverton-Holt model, the best values of the parameters that fit the data are $a = 1.7389$ and $b = 0.002165$, which gives the best fit model

$$P_{n+1} = \frac{1.7389P_n}{1 + 0.002165P_n}.$$

The least sum of square errors between the data and this model is 4637.79. One can easily see that the logistic model is only slightly better according to the sum of square errors.

b. The equilibria for the logistic model are $P_e = 0$ and $P_e = 337.36$. The derivative of this updating function is given by

$$f'(P) = 1.558 - 0.003308\,P.$$

At $P_e = 0$, $f'(0) = 1.558$, which shows that this equilibrium is unstable with solutions monotonically growing away from it. At $P_e = 337.36$, $f'(337.36) = 0.442$, which shows that this equilibrium is stable with solutions monotonically growing toward this equilibrium, the carrying capacity.

The equilibria for the Beverton-Holt model are $P_e = 0$ and $P_e = 341.29$. The derivative of this updating function is given by

$$B'(P) = \frac{1.7389}{(1 + 0.002165\,P)^2}.$$

At $P_e = 0$, $B'(0) = 1.7389$, which shows that this equilibrium is unstable with solutions monotonically growing away from it. At $P_e = 341.29$, $B'(341.29) = 0.5751$, which shows that this equilibrium is stable with solutions monotonically growing toward this equilibrium, the carrying capacity.

c. The discrete logistic growth model best fits the data with an initial population of $P_0 = 11.783$, which gives a sum of square errors of 3250.90. We find that the population at $t = 10$ weeks is $P_5 = 90.726$, the population at $t = 20$ weeks is $P_{10} = 294.61$, and the population at $t = 30$ weeks is $P_{15} = 336.40$.

The Beverton-Holt model best fits the data with an initial population of $P_0 = 7.606$, which gives a sum of square errors of 2938.36. We see that this best fitting initial condition is closer to the actual starting condition, and this model fits the time series data better according to the sum of square errors though not very much better. We find that the population at $t = 10$ weeks is $P_5 = 90.784$, the population at $t = 20$ weeks is $P_{10} = 290.82$, and the population at $t = 30$ weeks is $P_{15} = 337.61$. These simulations are very similar, not matching the data well at the beginning, but following the growth phase quite well, then reasonably matching what appears to be a carrying capacity around 340.

2. a. The solution for the simple model for pollution in Lake Erie is given by:

$$c(t) = k + (c_0 - k)e^{-\frac{35}{92}t}.$$

b. Using the data from the table, we find that the best fitting constants are $k = 4.8695$ and $c_0 = 2.0092$ with the sum of square errors being 0.031842. It follows that the best fitting solution is given by

$$c(t) = 4.8695 - 2.8603\, e^{-\frac{35}{92}\, t}.$$

c. The solution for this problem where we let the time at Year 5 be $t = 0$ is

$$c(t) = 4.5\, e^{-\frac{35}{92}\, t}.$$

It follows that the concentration of pollutant drops to half the amount in Year 5 when $t = 1.8220$ or less than two years later. $c(5) = 0.67160$ ppm and $c(10) = 0.10023$ ppm.

d. From the Euler solution, we find the approximate solution at $t = 1$ is $c(1) \simeq 4.41358$, $t = 2$ is $c(2) \simeq 4.16112$, $t = 3$ is $c(3) \simeq 3.82444$, $t = 5$ is $c(5) \simeq 3.08262$, $t = 7$ is $c(7) \simeq 2.39577$, and $t = 10$ is $c(10) \simeq 1.58579$. It is easy to see that these values are substantially higher than the ones in Part c.

e. From Maple's **dsolve**, the solution of the modified model for loss of pollution after a ban takes place is given by:

$$c(t) = 7.4292\, e^{-0.15\, t} - 2.9292\, e^{-\frac{35}{92}\, t}.$$

This modified model takes $t = 7.4766$ years for the pollutant level to fall to half the concentration in Year 5. The solution at $t = 10$ is $c(10) = 1.5924$. The percent error between the Euler approximation in Part c and the actual solution is given by

$$100 \frac{(1.58579 - 1.5924)}{1.5924} = -0.415\%.$$