1. Consider the points $(0,-3)$ and $(4,-1)$. Find the equation of the line (in point-slope form) passing through the two points. Also, find the equation of the line perpendicular to this line passing through the second of these points. Sketch a graph of these two lines.

2. Suppose that a 43 lb dog has a temperature of 102°F. Write the weight and temperature of this dog in kilograms and degrees Celsius. (Note that 1 kg = 2.2046 lb.)

3. Consider the functions:

$$f(x) = 2x - 1 \qquad \text{and} \qquad g(x) = 15 + 2x - x^2.$$

Find the coordinates of the x and y-intercepts for both functions. Find the slope of the line and the coordinates for the vertex of the parabola. Determine the coordinates for all points of intersection and sketch the graph.

4. Consider the functions $f(x) = x - 3$ and $g(x) = x^2 - 4x - 3$. Sketch the graphs of these functions. Include the coordinates of x and y-intercepts for both functions and the vertex of the parabola. Find the points of intersection.

5. The table below shows evaporation of water from a beaker. Initially, there is one liter. The loss by evaporation is linear. Find the equation of the line for V as a function of t. Determine when all the water is lost. Graph this function for all t when there is water in the beaker.

Time (t)	Volume (V)
0 week	1000 ml
1 week	940 ml
2 week	880 ml
4 week	760 ml

6. The following growth data were recorded for the height of a plant.

Week (t)	0	1	2	4
Height (cm) (h)	10.5	14	15.5	18

A linear model is proposed for the growth of this plant and has the form

$$h(t) = 2t + 11.$$

a. Find the sum of squares error between the data and the model. Sketch a graph of the model with the data points. Is the model reasonable?

b. Use the model to predict the height of the plant at 3 and 5 weeks.

7. For animals that reproduce seasonally, we find that their population satisfies a difference equation

$$P_{n+1} = P_n + g(P_n),$$

where P_n is the population in the nth season and $g(P)$ (in individuals per generation) is the growth rate of the population.

a. Suppose that the growth rate $g(P)$ satisfies the quadratic equation

$$g(P) = 0.1P\left(1 - \frac{P}{400}\right).$$

The population is at equilibrium when the growth rate is zero. Find the equilibrium populations.

b. The growth rate is at a maximum at the vertex of parabola. Find the population that produces this maximum growth rate and what that growth rate is.

c. Sketch a graph of this growth rate function.

8. If i $(0 \le i \le 1)$ is the fraction of infectious people in a community with an air-borne disease that imparts no immunity, then the fraction of susceptible people is $1 - i$. Assume that new infectious people are added at a rate $\alpha i(1 - i)$ with $\alpha = 0.1$ and infectious people are cured at a rate βi with $\beta = 0.07$. The rate of change in infectious people satisfies the function:

$$F(i) = 0.1i(1 - i) - 0.07i.$$

a. The disease is at equilibrium when the infection rate, $F(i)$, is zero. Find the equilibrium fractions of infectious people for this disease. (Hint: First rewrite $F(i)$ into standard factored quadratic form.)

b. The rate of spreading of the disease is at a maximum at the vertex of parabola. Find the fraction of infectious people, i, that produces this maximum rate of infection and what that infection rate is.

c. Find $F(1)$, then sketch a graph of this infection rate function, $F(i)$, $\quad 0 \le i \le 1$.

9. A rectangle with a length x and width y has a perimeter of 20 cm.

a. Write an expression for the width y as a function of the length x, using this information.

b. The area of a rectangle is $A = xy$. Substitute the expression for y into this formula for the area to produce a function of the area as a function of x alone. What is the domain of this function?

c. Sketch a graph of the area as a function of x and determine what value of x produces the largest area. What curve is produced by $A(x)$?

10. Suppose that $e^a = 2.2$ and $e^b = 0.7$. In addition, assume that $\ln(c) = 1.3$ and $\ln(d) = -0.5$. Use the properties of exponentials and logarithms to evaluate the following:

a. $\dfrac{e^{a+b}}{(e^b + e^0)^2}$

b. $\dfrac{(e^a)^2(e^0 - e^b)}{e^{2a+b}}$

c. $\dfrac{(\ln(c^3) - \ln(c) + \ln(1))}{(\ln(c) + \ln(e))}$,

d. $\dfrac{\ln(c^2 d) - \ln(1))}{(\ln(c/d) - \ln(e))}$

11. For each of the following functions, give the domain. Find the x and y-intercepts, and determine all vertical and horizontal asymptotes for each of these functions, then sketch a graph.

a. $y = x^3 - x^2 - 12x$

b. $y = \dfrac{50}{25 - x^2}$

c. $y = \dfrac{6x}{x+2}$

d. $y = \sqrt{16 - 2x}$

e. $y = 3x - 2x^2 - x^3$,

f. $y = 4 - \sqrt{5 - x}$,

g. $y = 20 - 5e^{-0.5x}$,

h. $y = 6\ln(5 - x) - 2$,

i. $y = \dfrac{4x}{2 + 0.001x}$,

j. $y = \dfrac{8x}{4 - x^2}$,

k. $y = 3 + 2\ln(x + 1)$,

l. $y = 6e^{x/2} - 2$,

m. $y = \dfrac{8x + 5}{6 - 2x}$,

12. Consider the trigonometric function:

$$y(x) = 5\sin(3x) - 4, \qquad x \in [0, 2\pi].$$

Find the amplitude, period, and vertical shift of this function. Give the x and y values for all maxima in the specified interval. Sketch the graph of this function.

13. a. Consider the trigonometric function:

$$y(x) = 2 - 4\cos(2x), \qquad x \in [0, 2\pi].$$

Find the amplitude, period, and vertical shift of this function. Give the x and y values for all maxima in the specified interval. Sketch the graph of this function.

b. Create an equivalent model in the form

$$y(x) = A + B\cos(\omega(x - \phi)),$$

where $B > 0$, $\omega > 0$, and $\phi \in [0, T)$ with T being the period of the function.

c. Create an equivalent model in the form

$$y(x) = C + D\sin(\nu(x - \psi)),$$

where $D > 0$, $\nu > 0$, and $\psi \in [0, T)$ with T being the period of the function.

14. Consider the trigonometric function:

$$y(t) = 7 - 4\cos\left(\frac{\pi}{8}(t - 5)\right), \qquad t \in [0, 20].$$

Find the period, amplitude, phase shift, and vertical shift of this function. Give the t and y values for all absolute maxima $(t_{max}, y(t_{max}))$ and absolute minima $(t_{min}, y(t_{min}))$ in the specified interval. (Note that there could be more than one maximum or minimum.) Sketch the graph of this function.

b. Create an equivalent model in the form

$$y(t) = A + B\cos(\omega(t - \phi)),$$

where $B > 0$, $\omega > 0$, and $\phi \in [0, T)$ with T being the period of the function.

c. Create an equivalent model in the form

$$y(t) = C + D\sin(\nu(t - \psi)),$$

where $D > 0$, $\nu > 0$, and $\psi \in [0, T)$ with T being the period of the function.

15. You are given the following data on the heights and lengths of several breeds of dogs. The height is measured at the shoulder (in cm) and the length is from the nose to the anus (in cm).

Breed	Height (H cm)	Length (L cm)
Chihuahua	18	33
Beagle	36	81
Labrador Retriever	55	102
Irish Setter	66	115

A linear model is proposed for the relationship between the length, L, and the height, H, of the following form:

$$L(H) = 1.7H + 10$$

a. Find the sum of squares error between the data and the model. Which breed is furthest from the model?

b. Use the model to predict the length of a Borzoi, which has a shoulder height of 81 cm. Also, use the model to predict the height of a Border Collie, which has a length of 85 cm.

16. Glucose regulation in the body is vital, and Type 1 diabetes occurs when this is disrupted by the loss of insulin producing cells. Below are data on glucose blood levels for a normal patient after ingesting a large amount of glucose at $t = 0$:

time, t (hr)	0.5	1.5	3
glucose, g (mg/dl)	150	90	80

a. A best fitting linear model for the level of glucose in the blood satisfies:

$$g(t) = 150 - 26\,t.$$

Use the data from the table above to write all the square errors, then compute the sum of square errors.

b. A better model for the level of glucose in the blood uses the exponential decay of metabolites:

$$G(t) = 80 + 180\,e^{-1.8t}.$$

Use the data from the table above with this model to write all the square errors, then compute the sum of square errors.

c. Consider the glucose model in Part b with $t \geq 0$. Find any t and G-intercepts in the **domain** and determine any horizontal or vertical asymptotes. Sketch the graph of this model.

17. The Lambert-Beer law for absorbance of light by a spectrophotometer is a linear relationship, which can have the form

$$A = mc,$$

where c is the concentration of the sample, A is absorbance, and m is the slope that must be determined from experiments.

a. Below are data collected on samples from a collection of urea standards using a urea indicator.

c (mM)	1	3	6
A	0.5	1.7	3.2

Write a formula for the quadratic function $J(m)$ that measures the sum of squares error of the line fitting the data. Find the vertex of this quadratic function. This gives the value of the best slope m, while the $J(m)$ value of the vertex gives the least sum of squares error.

b. Use this model (with the best value of m) to determine the concentration of urea in an unknown sample with absorbances of $A = 2.2$.

18. An underwater ecological study is made easier by photographing the region, then measuring distances on the picture. The picture is taken from above a flat rock reef. The diver measures three reference objects to help him with his study. One distinctive rock is 1.2 meters and measures 2.0 cm on the picture. Two kelp plants are separated by 2.5 meters, which is 4.0 cm on the picture. A sand bar is 3.6 meters across and measures 6.1 cm in the picture.

a. The conversion of measurements in the photo p to measurements in actual distance d is given by the formula

$$d = kp.$$

Write a formula for the quadratic function $J(k)$ that measures the sum of squares error of the line fitting the measurements in the photo. Find the vertex of this quadratic function. This gives the value of the best slope k, while the $J(k)$ value of the vertex gives the least sum of squares error.

b. In the photograph, there is a picture of a leopard shark that measures 2.2 cm. How long is this shark?

c. How long would a 2.0 m shark appear in the picture?

19. The poultry industry has accumulated detailed data on the consumption of feed by chickens. The reference *Nutritional Requirement of Chickens* (1984), you are given that a 560 g chicken consumes 390 g of feed per week. A 2520 g broiler consumes 1210 g of feed per week.

a. Assume linear relationship between the weight of the chicken (W) and the amount of feed (F) that it consumes

$$F = mW + b.$$

Use the data above to find the constants m and b in the model above.

b. Assume there is a power law relationship between the weight of the chicken (W) and the amount of feed (F) that it consumes

$$F = kW^a.$$

Use the data above to find the constants a and k in the power law or allometric model above.

c. Use both models (linear and allometric) to find the amount of feed consumed by a 1000 g chicken. Also, estimate the weight of a chicken that consumes 500 g of feed using both models. Which model gives the better predictions and why?

20. Experimental measurements show that when current is applied to samples of a tissue, the resistance measured by a voltmeter yields the thickness, T. Suppose a 3 mm sample of tissue causes a voltage drop, v, of 0.25V, and a 4 mm sample of tissue causes a voltage drop of 0.45V.

a. A linear model is given by $T = mv + b$ for some constants m and b. Find the constants m and b and sketch a graph of this model. Use this model to predict the voltage drop for a tissue that has a thickness of 2 mm. Also, find the thickness of a tissue that gives a voltage drop of 0.6V.

b. If the thickness of the tissue satisfies a power law with respect to resistance measured by the voltage drop, then the model is given by

$$T = kv^a,$$

Find the constants k and a and sketch a graph of this model. Use this model to predict the voltage drop for a tissue that has a thickness of 2 mm. Also, find the thickness of a tissue that gives a voltage drop of 0.6V.

c. Which model do you expect is better and why?

21. The muscles of the small intestine move chyme (food and enzymes) toward the colon in a process called *peristalsis* at a rate of 1-10 cm/sec. They periodically contract to create a traveling wave that causes the fluid in the small intestine to flow forward and allow absorption of nutrients into the blood. Consider a cross-section of the small intestine, assuming that it maintains a circular shape with a radius of $R(t)$ under the smooth muscle contraction. Suppose that the radius of one segment of the small intestine satisfies the function

$$R(t) = A + B\cos(\omega t),$$

where you must find the constants A, $B > 0$, and $\omega > 0$.

a. While digesting food, assume that the segment of small intestine periodically contracts 10 times per minute. Assume that the maximum distention of this segment occurs at $t = 0$ and is 4 cm ($R(0) = 4$). When the muscle is maximally contracted, the opening of the segment in the intestine reduces to only a radius of 1 cm (the minimum of $R(t)$). With these data, find the constants A, B, and ω.

b. Sketch the graph of $R(t)$ for $t \in [0, 0.2]$. List all maxima and minima in the interval.

c. Create an equivalent model of the form:

$$R(t) = C + D\sin(\nu(x - \psi)),$$

where $D > 0$, $\nu > 0$, and $\psi \in [0, T)$ with T being the period of the function.

22. In lab we considered a model for the length of day for various cities. The variation in length of day is quite pronounced when you go to Alaska.

a. In Anchorage, Alaska, the longest day is June 20 at 19 hr 22 min or 1162 min. The shortest day is December 21 at 5 hr 27 min or 327 min. Consider a model for the length of the day in minutes, $L(t)$, as a function of the date, t, using the sine function as follows

$$L(t) = \alpha + \beta \sin(\omega(t - \phi)),$$

where the constants α, $\beta > 0$, $\omega > 0$, and $\phi \in [0, 365)$ are to be determined below (and assuming that January 1 is $t = 0$).

Assume that June 20 is given by day 170 with length of 1162 min. With the information that the shortest day is 327 min and a year is 365 days, find the constants α, β, ω, and ϕ. Write the function $L(t)$ and find the length of Ground Hog day (February 2 or Day 32) in Anchorage.

b. Create an equivalent model of the form:

$$L(t) = C + D \cos(\nu(x - \psi)),$$

where $D > 0$, $\nu > 0$, and $\psi \in [0, 365)$.

23. Rattlesnakes are cold-blooded or ectothermic organisms with their body temperature depending on the external temperature. The body temperature of a rattlesnake tracks the external temperature with variations.

a. Suppose that the low temperature recorded for the rattlesnake on one day is 10°C at about 4 am, while the high is 26°C at about 4 pm. Assume that the body temperature for the rattlesnake can be modeled using the following function:

$$T(t) = A + B \sin(\omega(t - \phi)),$$

where A, $B > 0$, $\omega > 0$, and $\phi \in [0, 24)$ are constants and t is in hours. Use the data above to find the four parameters, then sketch a graph for the temperature of the rattlesnake for this day (midnight to midnight).

b. Create an equivalent model of the form:

$$R(t) = C + D \sin(\nu(x - \psi)),$$

where $D > 0$, $\nu > 0$, and $\psi \in [0, 24)$.

24. a. A population of herbivores satisfies the growth equation

$$H_{n+1} = 1.02 H_n.$$

If the initial population is $H_0 = 2000$, then determine the populations H_1 and H_2. Also, give an expression for the population H_n in terms of H_0 and n.

b. Another group of herbivores satisfies the growth equation

$$G_{n+1} = 1.03 G_n.$$

If the initial population is $G_0 = 200$, then give an expression for the population G_n in terms of G_0 and n. Determine how long does it take for this population to double.

c. Find when the two populations are equal.

25. The population of the United States was about 179.3 million in 1960 and 226.5 million in 1980. Let 1960 be represented by P_0 and assume that its population is growing according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is in years.

a. Use the data above to find the annual growth rate r, then write an expression for the population in any year following 1960. (Write the solution P_n in terms of P_0 with n being the number of years after 1960.)

b. Predict the population in the year 2000. The actual population was about 281.4 million. What is the error between the model and the actual census data?

c. According to the model, how long until the U. S. population doubles from its 1960 level?

26. a. The population of the France in 1980 was about 53.9 million, and a census in 1990 showed that the population had grown to 56.7 million. Assume that this population grows according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1980, and P_n is population n decades after 1980. Use the data above to find the growth constant r, then write the general solution P_n.

b. Predict the population in the years 2000 and 2020. France's population in 2000 was 59.4 million. Use this information to compute the percent error between the Malthusian growth model and the actual census data.

c. In 1980, the population of Kenya was 16.7 million, while in 1990, it had grown to 24.2 million. Assume its population is also growing according to a Malthusian growth law. Find its rate of growth per decade and predict its population in 2000 and 2020. How long does it take for Kenya's population to double?

d. If these countries continue to grow according to these Malthusian growth laws, then determine the first year when Kenya's population will exceed that of the France and determine their populations at that time.

e. Find the annual growth rate for both France and Kenya between 1980 and 1990.

27. a. The population of Poland in 1950 was about 24.82 million, while in 1970, it was about 32.53 million. Assume that the population is growing according to the discrete Malthusian growth equation

$$P_{n+1} = (1 + r)P_n, \quad \text{with} \quad P_0 = 24.82,$$

where P_0 is the population in 1950 and n is in years. Use the population in 1970 to find the **annual growth rate**, r (to **4 significant figures**). Find how long it would take for this population to double.

b. Estimate the population in 2000 based on the Malthusian growth model. Given that the population in 2000 was 38.65 million, find the percent error between the actual and predicted values. (Assume the actual census data is the best value.)

c. A better model fitting the census data for Poland is a logistic growth model given by

$$P_{n+1} = P_n + G(P_n) = P_n + 0.042P_n - 0.001P_n^2,$$

where again n is in years after 1950. The annual population growth function satisfies:

$$G(P) = 0.042P - 0.001P^2,$$

where P is the population in millions. The equilibria for the logistic growth model occur when $G(P_e) = 0$. Find the equilibria.

d. The fastest growth rate occurs at the vertex of the growth function $G(P_v)$. Find the population and rate of growth when Poland's population is growing most rapidly.

28. a. Consider a model with immigration given by

$$p_{n+1} = 0.8p_n + 300,$$

with an initial population of $p_0 = 500$. Determine the populations at the next three time intervals, p_1, p_2, and p_3.

b. Find all equilibria and determine the stability of these equilibria.

29. A man with a chronic lung problem has a tidal volume, V_i, of 300 ml. For this experiment, Helium, He, is used to determine the functional reserve capacity, V_r. (Note that $V_r = (1-q)V_i/q$.) The mathematical model gives

$$c_{n+1} = (1-q)c_n + q\gamma,$$

where $\gamma = 5.2$ ppm.

a. The man is given an enriched mixture of air to breathe that contains 50 ppm of He. Experimentally, the concentration of He in the first two measured breaths after breathing the enriched mixture are given by $c_0 = 50$ and $c_1 = 44.6$ ppm. Use c_0 and c_1 to find q, then determine the functional reserve capacity, V_r.

b. Use your model to find the expected concentration of Helium in this patient's 3^{rd} breath, c_3. What is the equilibrium concentration of Helium in the patient's lungs? What is the stability of this equilibrium concentration?

30. Below are data on the population of insect pests living in a survey area. The insect reproduces according to a Malthusian growth model and disperses (emigrates) to surrounding regions at a constant rate. The population model for this insect pest is given by

$$P_{n+1} = (1+r)P_n - \mu,$$

where r is the rate of growth (per week) and μ is the number of pests dispersing each week to surrounding regions.

a. From the data below determine the updating function for this population, *i.e.*, find r and μ. Then use this updating function to find the population of the insect pests for weeks 3 and 4.

b. Find all equilibria for this model. Based on your iterations in Part a, what is the stability of the equilibria? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $P_{n+1} = P_n$. Determine all points of intersection.

Week	Insects
0	500
1	630
2	825

31. A woman with a chronic lung problem breathes a supply of air enriched with Helium. Experimental measurements show the following concentrations of the exhaled air after she resumes normal breathing in the room.

Breath Number	0	1	2
Conc. of He (ppm)	400	352	310

The concentration of Helium in the room, γ, is not known, but assumed to be constant.

a. Assume a breathing model of the form:

$$c_{n+1} = (1 - q)c_n + q\gamma.$$

Use the data above to find the constants q, the fraction of air exchanged, and γ, the ambient concentration of Helium. Then determine the concentration of Helium in the next two breaths, c_3 and c_4.

b. Find the equilibrium concentration of Helium in the subject's lungs based on this breathing model. What is the stability of this equilibrium concentration? (If a solution moves closer to an equilibrium point, then it is probably stable. If it moves away, then it is most likely unstable.)

c. Graph the updating function along with the identity map, $c_{n+1} = c_n$, showing all intercepts ($c_n \geq 0$) and points of intersection.