1. a. The best model for the amount of drug in the body is

$$D(t) = 7.7784e^{-0.1077t}.$$

The sum of square errors between the model and the data is 0.78767.

b. The best parameters fitting the data are $B = 297.97$ and $q = 0.07603$, giving the best pharmakinetic model as

$$C(t) = 297.97(e^{-0.07603t} - e^{-0.1077t}).$$

The sum of square errors between the model and the data is 1.0517. With this exponential model, the derivative is

$$C'(t) = -22.655e^{-0.07603t} + 32.091e^{-0.1077t}.$$

The percent error at $t = 10$ days is -1.007%, while the percent error at $t = 20$ days is -0.1299%. The maximum concentration of the cytokine occurs at $t_{max} = 10.995$ with $C(t_{max}) = 37.979$ ng/dl. See graph below.

c. The best parameters fitting the data are $K = 9.3975$ and $r = 0.09090$, giving the best pharmakinetic model as

$$R(t) = 9.3975te^{-0.09090t}.$$

The sum of square errors between the model and the data is 1.0137. This gives the smaller sum of square errors, but only by an insignificant amount, making the models equally good. The percent error at $t = 10$ days is -0.8768%, while the percent error at $t = 20$ days is -0.2820%. With this Ricker's model, the derivative is

$$R'(t) = 9.3975\,e^{-.09090t}(1 - 0.09090t).$$

From this model, the maximum concentration of the cytokine occurs at $t_{max} = 11.001$ with $R(t_{max}) = 38.032$ ng/dl. See graph below.
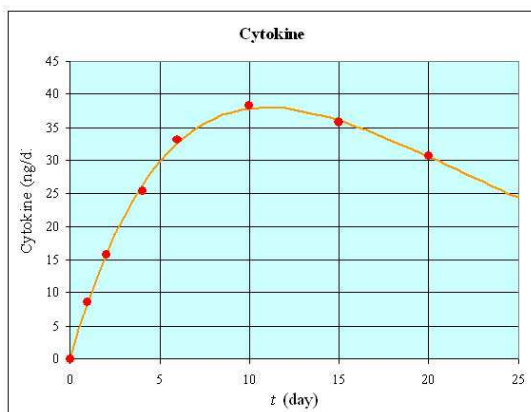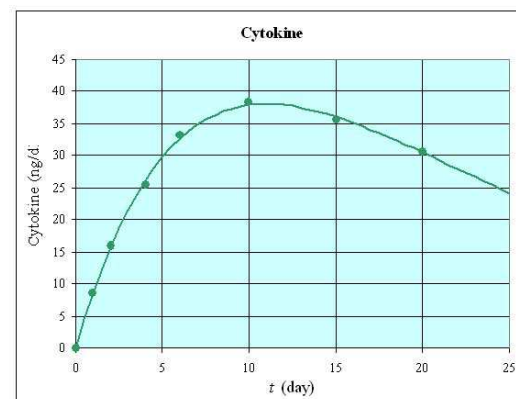
$C(t)$ from Part b $R(t)$ from Part c

2. a. With the model given by

$$G(t) = G_0 + Ae^{-\alpha t}\cos(\omega(t - \delta)),$$

the first set of data gives the best fitting parameters $G_0 = 83.893$, $A = 175.813$, $\alpha = 0.9133$, $\omega = 1.87045$, and $\delta = 0.87294$. It follows that best model is:

$$G_1(t) = 83.893 + 175.813\,e^{-0.9133t}\cos(1.87045(t - 0.87294)).$$

The sum of square errors is 303.108.

The second set of data gives the best fitting parameters $G_0 = 106.075$, $A = 207.729$, $\alpha = 0.4934$, $\omega = 1.1133$, and $\delta = 1.4214$. It follows that best model is:

$$G_2(t) = 106.075 + 207.729\,e^{-0.4934t}\cos(1.1133(t - 1.4214)).$$

The sum of square errors is 781.406.

b. Below are graphs of the data for the models fitting these patients. The absolute maximum for $G_1(t)$ is $t_{max} = 0.63010$ hr with $G_1(t_{max}) = 172.751$ mg/dl of blood, while the absolute minimum is $t_{min} = 2.3097$ hr with $G_1(t_{min}) = 64.728$ mg/dl of blood. The absolute maximum for $G_2(t)$ is $t_{max} = 1.0467$ hr with $G_2(t_{max}) = 219.38$ mg/dl of blood, while the absolute minimum is $t_{min} = 3.8686$ hr with $G_1(t_{min}) = 77.918$ mg/dl of blood.
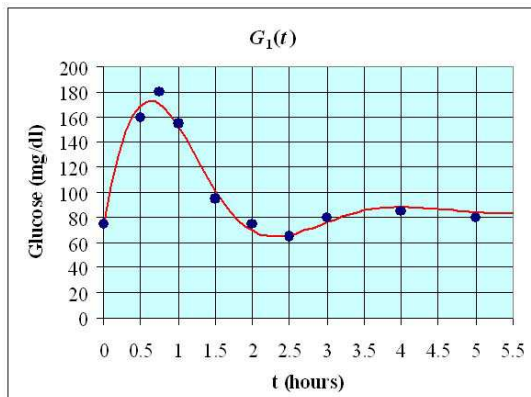
$G_1(t)$ and data
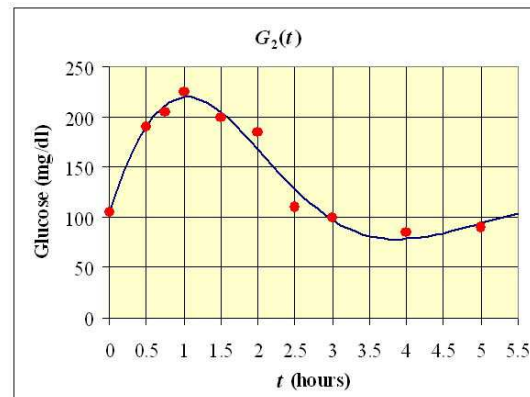
$G_1(t)$ and data

c. From the model for the first patient, $G_1(t)$, $\omega_0 = \sqrt{\omega^2 + \alpha^2} = 2.0815$. Since $2\pi/\omega_0 = 3.0186 < 4$, this patient is normal. From the model for the second patient, $G_2(t)$, $\omega_0 = \sqrt{\omega^2 + \alpha^2} = 1.218$. Since $2\pi/\omega_0 = 5.160 > 4$, this patient is diabetic.