1. a. With $n = 2$ and $x \in [0, 2]$, the midpoints of the subintervals are $x_1 = \frac{1}{2}$ and $x_2 = \frac{3}{2}$ with $\Delta x = 1$, so the midpoint rule gives

$$\int_0^2 (4 + 2x^2)\, dx \approx \left(\left(4 + 2\left(\frac{1}{2}\right)^2\right) + \left(4 + 2\left(\frac{3}{2}\right)^2\right)\right) \cdot 1 = 13.$$

With $n = 2$, the trapezoid rule gives

$$\int_0^2 (4 + 2x^2)\, dx \approx \left(\frac{1}{2}\left(4 + 2(0)^2\right) + \left(4 + 2(1)^2\right) + \frac{1}{2}\left(4 + 2(2)^2\right)\right) \cdot 1 = 14.$$

b. With $n = 4$ and $x \in [0, 2]$, the midpoints of the subintervals are $x_1 = \frac{1}{4}$, $x_2 = \frac{3}{4}$, $x_3 = \frac{5}{4}$, $x_4 = \frac{7}{4}$, with $\Delta x = \frac{1}{2}$, so the midpoint rule gives

$$\begin{aligned} \int_0^2 (4 + 2x^2)\, dx &\approx \left(\left(4 + 2\left(\frac{1}{4}\right)^2\right) + \left(4 + 2\left(\frac{3}{4}\right)^2\right) + \left(4 + 2\left(\frac{5}{4}\right)^2\right) + \left(4 + 2\left(\frac{7}{4}\right)^2\right)\right) \cdot \frac{1}{2} \\ &= 13.25. \end{aligned}$$

With $n = 4$, the trapezoid rule gives

$$\begin{aligned} \int_0^2 (4 + 2x^2)\, dx &\approx \left(\frac{1}{2}\left(4 + 2(0)^2\right) + \left(4 + 2\left(\frac{1}{2}\right)^2\right) + \left(4 + 2(1)^2\right)\right. \\ &\quad \left. + \left(4 + 2\left(\frac{3}{2}\right)^2\right) + \frac{1}{2}\left(4 + 2(2)^2\right)\right) \cdot \frac{1}{2} = 13.5. \end{aligned}$$

c. For $n = 2$, the midpoint rule has a percent error of

$$100\left(\frac{13 - \frac{40}{3}}{\frac{40}{3}}\right) = -2.5\% \text{ error,}$$

which is a low estimate.

For $n = 2$, the trapezoid rule has a percent error of

$$100\left(\frac{14 - \frac{40}{3}}{\frac{40}{3}}\right) = 5.0\% \text{ error,}$$

which is a high estimate.

Similarly, for $n = 4$, the midpoint rule has a percent error of

$$100\left(\frac{13.25 - \frac{40}{3}}{\frac{40}{3}}\right) = -0.625\% \text{ error,}$$

which is a low estimate.

The trapezoid rule has a percent error of

$$100\left(\frac{13.5-\frac{40}{3}}{\frac{40}{3}}\right)=1.25\%\text{ error},$$

which is a high estimate.

2. a. With $n=4$ and $x\in[0,2]$, the midpoints of the subintervals are $x_1=\frac{1}{4}$, $x_2=\frac{3}{4}$, $x_3=\frac{5}{4}$ and $x_4=\frac{7}{4}$ with $\Delta x=\frac{1}{2}$, so the midpoint rule gives

$$\begin{aligned}\int_0^2 x^4\,dx &\approx \left(\left(\frac{1}{4}\right)^4+\left(\frac{3}{4}\right)^4+\left(\frac{5}{4}\right)^4+\left(\frac{7}{4}\right)^4\right)\cdot\frac{1}{2}\\ &= 6.0703125.\end{aligned}$$

With $n=4$, the trapezoid rule gives

$$\begin{aligned}\int_0^2 x^4\,dx &\approx \left(\frac{1}{2}(0)^4+\left(\frac{1}{2}\right)^4+(1)^4+\left(\frac{3}{2}\right)^4+\frac{1}{2}(2)^4\right)\cdot\frac{1}{2}\\ &= 7.0625.\end{aligned}$$

With $n=4$, Simpson's rule gives

$$\begin{aligned}\int_0^2 x^4\,dx &\approx \left((0)^4+4\left(\frac{1}{2}\right)^4+2(1)^4+4\left(\frac{3}{2}\right)^4+(2)^4\right)\cdot\frac{1}{2\cdot 3}\\ &= \frac{154}{24}\approx 6.416667.\end{aligned}$$

b. For $n=4$, the midpoint rule has a percent error of

$$100\left(\frac{6.07031-\frac{32}{5}}{\frac{32}{5}}\right)=-5.15\%\text{ error},$$

which is a low estimate.

The trapezoid rule has a percent error of

$$100\left(\frac{7.0625-\frac{32}{5}}{\frac{32}{5}}\right)=10.35\%\text{ error},$$

which is a high estimate.

Simpson's rule has a percent error of

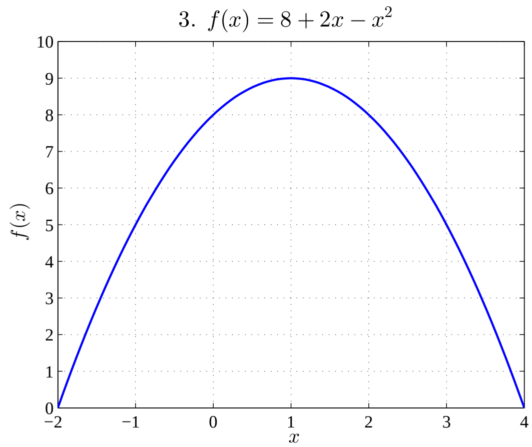
$$100\left(\frac{6.4167-\frac{32}{5}}{\frac{32}{5}}\right)=0.26\%\text{ error},$$

which is a high estimate, but very close.

3. a. At the y-intercept, $x = 0$, and $f(0) = 8$. The x -intercepts occur where

$$8 + 2x - x^2 = 0 \qquad \text{or} \qquad (4 - x)(2 + x) = 0$$

or the x-intercepts are $(-2, 0)$, $(4, 0)$. The vertex is midway between the two points at $x = 1$, when $f(1) = 8 + 2 - 1 = 9$. The graph is shown below.



b. With $n = 4$ and $x \in [0, 4]$, the midpoints of the subintervals are $x_1 = \frac{1}{2}$, $x_2 = \frac{3}{2}$, $x_3 = \frac{5}{2}$ and $x_4 = \frac{7}{2}$ with $\Delta x = 1$, so the midpoint rule gives

$$\begin{aligned}\int_0^4 \left(8 + 2x - x^2\right)\,dx &\approx \left(\left(8 + 2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)^2\right) + \left(8 + 2\left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2\right)\right.\\ &\quad \left. + \left(8 + 2\left(\frac{5}{2}\right) - \left(\frac{5}{2}\right)^2\right) + \left(8 + 2\left(\frac{7}{2}\right) - \left(\frac{7}{2}\right)^2\right)\right) \cdot 1 = 27.\end{aligned}$$

With $n = 4$, the trapezoid rule gives

$$\begin{aligned}\int_0^4 \left(8 + 2x - x^2\right)\,dx &\approx \left(\frac{1}{2}(8) + \left(8 + 2(1) - (1)^2\right) + \left(8 + 2(2) - (2)^2\right)\right.\\ &\quad \left. + \left(8 + 2(3) - (3)^2\right) + \frac{1}{2}\left(8 + 2(4) - (4)^2\right)\right) \cdot 1 = 26.\end{aligned}$$

c. With $n = 4$, Simpson's rule gives

$$\begin{aligned}\int_0^4 \left(8 + 2x - x^2\right)\,dx &= \left((8) + 4\left(8 + 2(1) - (1)^2\right) + 2\left(8 + 2(2) - (2)^2\right)\right.\\ &\quad \left. +4\left(8 + 2(3) - (3)^2\right) + \left(8 + 2(4) - (4)^2\right)\right) \cdot \frac{1}{3} = \frac{80}{3} \approx 26.667.\end{aligned}$$

For $n = 4$, the midpoint rule has a percent error of

$$100\left(\frac{27-\frac{80}{3}}{\frac{80}{3}}\right) = 1.25\% \text{ error},$$

which is a high estimate.

The trapezoid rule has a percent error of

$$100\left(\frac{26-\frac{80}{3}}{\frac{80}{3}}\right) = -2.5\% \text{ error},$$

which is a low estimate.

4. a. The average population is

$$\frac{12+18+27+32+28+17+12+21}{8} = \frac{167}{8} \approx 20.875.$$

b. The trapezoid rule with $n = 7$, and $\Delta x = 1$

$$P_{ave} = \frac{1}{7}\int_0^7 P(t)\ dt = \frac{1}{7}\left(\frac{1}{2}(12)+18+27+32+28+17+12+\frac{1}{2}(21)\right)\cdot 1 = 21.5.$$

The answer is slightly higher as the endpoints are not weighed as heavily.

5. The trapezoid rule with $n = 10$, and $\Delta x = 1$

$$\begin{aligned} A_{cum} &= \int_0^{10} A(t)\ dt. \\ &= \left(\frac{1}{2}(0.05)+0.46+0.87+0.54+0.43+0.36+0.28+0.21+0.16+0.12+\frac{1}{2}(0.09)\right)\cdot 1 \\ &= 3.5. \end{aligned}$$