Integrate the following:

1. $\int x\sqrt{2x^2-3}\,dx,$
2. $\int \frac{4}{2x-6}\,dx,$
3. $\int \frac{x+2}{(x^2+4x-5)^3}\,dx,$
4. $\int x\sin(x^2+4)\,dx,$
5. $\int \frac{(x-1)}{e^{x^2-2x}}\,dx,$
6. $\int \frac{5\sin(2x)}{(\cos(2x)+6)}\,dx,$
7. $\int \frac{e^{\sqrt{x+7}}}{\sqrt{x+7}}\,dx,$
8. $\int \frac{x^3}{\sqrt{1-x^4}}\,dx.$

Solve the following initial value problems. (Note some problems may use techniques from previous sections.)

9. $\frac{dy}{dt}=t\sqrt{t^2+1}, \qquad y(0)=5,$
10. $\frac{dy}{dt}=t\sin(t^2-4), \qquad y(2)=3,$
11. $\frac{dy}{dt}=\frac{ty}{\sqrt{t^2-1}}, \qquad y(1)=4,$
12. $\frac{dy}{dt}=0.1t(4-y), \qquad y(0)=10,$
13. $t\frac{dy}{dt}=2\left(\ln(t)\right)^4, \qquad y(1)=3.$

14. A culture of yeast is growing in a limited medium. There are initially 1000 yeast. The culture grows according to the logistic growth equation

$$\frac{dP}{dt}=0.2P\left(1-\frac{P}{50,000}\right),$$

where t is in hours. Find the general solution to this equation. Determine how long it takes for the original population to double. Also, find how long it takes for the culture to reach half its carrying capacity.

15. Growing cells absorb their nutrients through their surface, which indicates that a volume growth model should be proportional to the 2/3 power. In addition, if a cell is in an environment with competition, the growth rate would decline with time. Thus, a growth law for a growing cell in culture is written

$$\frac{dV}{dt}=k(t)V^{2/3}, \qquad V(0)=1.$$

a. Suppose that a growth rate is measured to be

$$k(t)=\frac{0.12t}{t^2+1},$$

where t is in minutes and includes both the initial lag time of growth and later slowing from competition. Find the maximum growth rate, $k(t_{max})$ and the time, t_{max}, when it occurs. You should sketch a graph of $k(t)$.

b. Solve the differential equation above with the given growth rate, $k(t)$.

c. Find how long it takes for this cell to double in volume.