

1. A. C. Crombie studied *Oryzaephilus surinamensis*, the saw-tooth grain beetle, with an almost constant nutrient supply (maintained 10 g of cracked wheat weekly). These conditions match the assumptions of the discrete logistic model. The data below show the adult population of *Oryzaephilus* from Crombie's study (with some minor modifications to fill in uncollected data and an initial shift of one week).

Week	Adults	Week	Adults
0	4	16	405
2	4	18	471
4	25	20	420
6	63	22	430
8	147	24	420
10	285	26	475
12	345	28	435
14	361	30	480

The discrete logistic growth model for the adult population P_n can be written

$$P_{n+1} = f(P_n) = rP_n - mP_n^2,$$

where the constants r and m must be determined from the data.

a. Plot P_{n+1} vs. P_n , which you can do by entering the adult population data from times 2–30 for P_{n+1} and times 0–28 for P_n . (Be sure that P_n is on the horizontal axis.) To find the appropriate constants use Excel's Trendline with its polynomial fit of order 2 and with the intercept set to 0 (under options). In your lab, write the equation of the model which fits the data best. Graph both $f(P)$ and the data.

b. Find the equilibria for this model. Write the derivative of the updating function. Discuss the behavior of the model near its equilibria. (Note that if P_e is an equilibrium point, then you can determine the behavior of that equilibrium by evaluating the derivative of the updating function $f(P_n)$ at P_e . Simulate the model and show this simulation compared to the data from the table above (adult population vs. time). Use the values that you found above for r and m , then perform a least squares best fit of the initial population P_0 to the time series data. Report the best P_0 and the sum of square errors. Discuss how well your simulation matches the data in the table. What do you predict will happen to the adult saw-tooth grain beetle population for large times (assuming experimental conditions continue)?

c. Another common population model is Ricker's, which is given by

$$P_{n+1} = R(P_n) = aP_n e^{-bP_n},$$

where a and b are constants to be determined. Use Excel's solver to find the least squares best fit of the Ricker's updating function to the given data by varying a and b . As initial guesses take $a = 2.5$ and $b = 0.002$. Once again plot P_{n+1} vs. P_n , using this updating function and show how it compares to the data (much as you did in Part a.

d. Find the equilibria for Ricker's model. Write the derivative of the updating function, then discuss the behavior of these equilibria using this derivative. (Give the value of the derivative at the equilibria.) Simulate the discrete dynamical system using Ricker's model. Use the

values that you found above for a and b , then perform a least squares best fit of the initial population P_0 to the time series data. Report the best P_0 and the sum of square errors and compare this to the one found above for the logistic growth model. Show the graphs of the logistic and Ricker's models with the data. Compare these simulations with the data. Discuss the similarities and differences that you observe between models and how well they work for this experimental situation.