1. Below are data on the population of a species of moth that inhabits an island and breeds annually (then dies). If its offspring have a survival rate r, and there is a net (constant) influx of new moths from surrounding islands entering at a rate μ, then the population model has the form

$$P_{n+1} = rP_n + \mu.$$

a. From the data below determine the updating function for this population, *i.e.*, find r and μ. Then use this updating function to find the population of moths in 1993, 1994, and 1995. Write a closed form solution to this problem, *i.e.*, a solution P_n that depends only on P_0, r, μ, and n.

b. Find all equilibria for this model. Based on your iterations in Part a, what is the stability of the equilibria? What does this model predict will ultimately happen to the population of moths?

c. Graph the updating function along with the identity map, $P_{n+1} = P_n$. Determine all points of intersection.

Year	Moths
1990	6000
1991	5500
1992	5100

2. a. Hassell's model is often used to study populations of insects. The genereal model is given by

$$P_{n+1} = H(P_n) = \frac{aP_n}{(1+bP_n)^c},$$

where $a > 1$, b, and c are positive parameters. Find all equilibria for this general model.

b. Compute $H'(P)$, then evaluate this derivative at the equilibria. What is the stability of the smallest equilibrium? (Recall $a > 1$.) Describe the possible qualitative behaviors of the largest equilibrium when $c = 1$, 2, and 4.

3. In 1946, A. C. Crombie studied a number of populations of insects with the amount of food supplied strictly regulated. One study examined *Oryzaephilus surinamensis*, the saw-tooth grain beetle. The population data are given in the table below:

Week	Population
0	2
2	2
4	4
5	33
6	41
7	53
9	74
11	127
13	190
15	203
17	305
19	385
21	480
23	405
25	425
27	425
29	450
33	415
37	425
39	415
41	420

a. The experiments were designed to satisfy the conditions for logistic growth. The discrete logistic growth equation is given by

$$P_{n+1} = F(P_n) = P_n + rP_n\left(1 - \frac{P_n}{M}\right),$$

where n is in weeks. From the data above, find the best values of the parameters P_0, M, and r, using the least squares best fit to the data. Write the values of these parameters and the formula for the discrete logistic growth model that best fits the saw-tooth grain beetle population in the study above. Also, give the value of the sum of square error between the data and the model with the best fitting parameters. Use Excel to create a graph that shows the data (as points only) and the best fitting model (as straight lines connecting each week of simulated data). Record the simulated populations for weeks 5, 10, 15, 20, 25, and 35.

b. For the discrete logistic growth model found above, determine all equilibria. Use the techniques from class to determine the stability of these equilibria, giving the values of the derivatives at each of the equilibria.

c. Above we noted that Hassell's model is usually applied to insect populations. Apply Hassell's model in the form:

$$P_{n+1} = H(P_n) = \frac{aP_n}{(1 + bP_n)^c},$$

where n is in weeks. From the data above, find the best values of the parameters P_0, a, b, and c, using the least squares best fit to the data. Write the values of these parameters and the formula for Hassell's growth model that best fits the saw-tooth grain beetle population in the study above. Again, give the value of the sum of square error between the data and the model with the best fitting parameters. (Note that you may want to use the option on Excel's solver that forces the parameters to be positive.) Which model better fits the data according to the sum of square error? Use Excel to create another graph that shows the data (as points only) and the best fitting Hassell's model (as straight lines connecting each week of simulated data). Record the simulated populations from Hassell's model for weeks 5, 10, 15, 20, 25, and 35.

d. For Hassell's growth model, determine all equilibria. Use the techniques from class to determine the stability of these equilibria, giving the values of the derivatives at each of the equilibria.

e. Take the data for *Oryzaephilus surinamensis*, the saw-tooth grain beetle, and plot P_{n+1} vs. P_n. To this graph add the identity map and the updating functions $F(P_n)$ (logistic growth function) and $H(P_n)$ (Hassell's growth function). Graph the functions for $0 \leq P_n \leq 1000$ with range $0 \leq P_{n+1} \leq 500$. How do these functions compare, especially near the data? Find the P_n-intercepts for both $F(P_n)$ and $H(P_n)$. Find the maximum for both $F(P_n)$ and $H(P_n)$. Do either of these functions have vertical or horizontal asymptotes? List the asymptotes.

4. The populations of 12 groups of birds were surveyed over 5 years. The researchers measured the birth rates, death rates, and recruitment rates for each of the 12 survey groups. The bird population is assumed to follow the model

$$P_{n+1} = (1 + b_i - d_i)P_n + r_i.$$

Below is a table summarizing their results (and these data can be acquired from a hyperlink on the webpage).

a. Compute the mean, median, and variance for the parameters b_i, d_i, and r_i. Create a histogram of the death rate, d_i, data making the bins a width of 0.05. What distribution best matches these data? Also, create a box-plot for the data on the recruitment values, r_i.

b. Starting with $P_0 = 200$, use the mean values of b_i, d_i, and r_i to simulate the bird population for 5 years. Find the equilibria for this model and determine their stability. Repeat the simulation and qualitative analysis (equilibria and stability) using the median values of b_i, d_i, and r_i instead.

c. Starting with $P_0 = 200$, use the data in the table below to find the population for each of the 5 years for Group 5 and Group 10.

Year		Group 1			Group 2			Group 3	
	b_i	d_i	r_i	b_i	d_i	r_i	b_i	d_i	r_i
1	0.18	0.34	46	0.10	0.35	58	0.22	0.41	37
2	0.30	0.26	55	0.30	0.31	35	0.34	0.02	54
3	0.03	0.36	40	0.18	0.24	55	0.29	0.53	40
4	0.11	0.36	49	0.14	0.44	44	0.06	0.33	47
5	0.05	0.26	62	0.12	0.18	62	0.14	0.52	53
		Group 4			Group 5			Group 6	
	b_i	d_i	r_i	b_i	d_i	r_i	b_i	d_i	r_i
1	0.03	0.28	37	0.15	0.64	36	0.17	0.07	36
2	0.21	0.38	39	0.17	0.15	41	0.17	0.07	51
3	0.17	0.21	37	0.31	0.18	47	0.19	0.46	45
4	0.19	0.23	32	0.16	0.10	50	0.36	0.56	32
5	0.27	0.17	44	0.39	0.40	34	0.15	0.47	62
		Group 7			Group 8			Group 9	
	b_i	d_i	r_i	b_i	d_i	r_i	b_i	d_i	r_i
1	0.33	0.26	60	0.07	0.30	42	0.24	0.51	50
2	0.27	0.47	48	0.35	0.34	40	0.19	0.08	46
3	0.16	0.37	43	0.25	0.46	42	0.17	0.40	36
4	0.26	0.43	51	0.08	0.56	37	0.13	0.26	51
5	0.08	0.53	37	0.17	0.31	47	0.20	0.68	52
		Group 10			Group 11			Group 12	
	b_i	d_i	r_i	b_i	d_i	r_i	b_i	d_i	r_i
1	0.20	0.38	36	0.17	0.42	42	0.38	0.45	38
2	0.06	0.20	42	0.14	0.22	49	0.01	0.58	51
3	0.32	0.45	42	0.10	0.40	44	0.15	0.58	41
4	0.26	0.49	35	0.24	0.15	40	0.27	0.43	43
5	0.25	0.40	43	0.09	0.43	34	0.11	0.19	46