Give all answers to at least **4 significant figures**.

1. The growth of the world population has begun to slow for a number of reasons, including increased death from diseases such as AIDS and improved family planning options for women. Below is a table showing the world population (in billions) for the last half of the 20^{th} century.

Year	1950	1960	1970	1980	1990	2000
Population (in billions)	2.556	3.039	3.707	4.454	5.279	6.083

a. A Malthusian growth model with a time-varying growth rate provides an appropriate model for human population growth, as seen in the lecture notes. The time-varying Malthusian growth model is given by the following differential equation:

$$\frac{dP}{dt} = (b - at)P, \qquad P(0) = P_0,$$

where t is in years after 1950. (Let $t = 0$ be 1950.) This model has a simple decreasing linear growth rate with Malthusian growth rate b. Find the general solution to this differential equation, using the parameters a, b, and P_0.

b. Find the least squares best fit parameters for this model to the world population data above. Write the values of the best parameters a, b, and P_0 to fit the data above. Also, give the sum of squares error after applying Excel's solver routine.(You should apply Solver at least twice to guarantee it converges.) Graph the data and the best fitting model up to the year 2050.

c. With the best fitting model, give the projected populations of the world population in the years 2000, 2050, and 2100. What year does the model predict that the population of the world will reach its maximum and list that maximum population?

2. When a monoculture of an organism is grown in a limited (but renewed) medium, then the population of that organism often follows the logistic growth model. Below is a table from a study by G. F. Gause [1] where he grew cultures of *Paramecium aurelia* and counted the number of individuals/0.5 cc.

Day	0	2	3	4	5	6	7	8	9	10	11	12
P. aurelia	2	14	34	56	94	189	266	330	416	507	580	610

a. Consider the Malthusian growth model

$$\frac{dP}{dt} = rP, \qquad P(0) = P_0.$$

Write the general solution to this model.

b. Find the best Malthusian growth model that fits the first **six** days (0-6) of the experiment, using Excel's Trendline with the exponential fitting option. Give the value of the best fitting parameters r and P_0. Show the graph of these six days of data and the Malthusian growth model.

Write the population given by the model at days $t = 3$, 5, and 8. Compute the percent error between the model and data at days $t = 3$, 5, and 8.

c. As noted above, these experiments are best designed to fit the logistic growth model, which is given by

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{M}\right), \qquad P(0) = P_0,$$

where r is the Malthusian growth rate and M is the carrying capacity. Use Maple to find the general solution to this equation (with the parameters r, M, and P_0.)

d. Use a nonlinear least squares best fit to the data to fit the general solution of the logistic growth model to the data in the table. List the best fit values for the parameters r, M, and P_0. Also, give the sum of squares error after applying Excel's solver routine. Graph the data and the best fitting model up to $t = 15$ hours. Write the population given by the logistic growth model at days $t = 3$, 5, 8, and 11. Compute the percent error between the model and data at days $t = 3$, 5, 8, and 11. If the experimental conditions are continued, then what will be the population of *P. aurelia* after a long period of time?

[1] G. F. Gause (1934), *Struggle for Existence*, Hafner, New York.