

Write your project solutions in a **Word** document. (Do **NOT** turn in lists of numbers from Excel worksheets.) Simply answer all questions below in complete sentences. Points will be taken off for not following directions. Give all answers to at least **4 significant figures**.

1. In 1946, A. C. Crombie studied a number of populations of insects with the amount of food supplied strictly regulated. One study examined *Rhizopertha dominica*, the lesser grain borer. The population data are given in the table below:

Week	Population
0	2
2	2
4	2
5	3
6	17
7	65
9	119
11	130
13	175
15	205
17	261
19	302
21	330
23	315
25	333
27	350
29	332
33	333
35	335
37	330

a. The experiments were designed to satisfy the conditions for logistic growth. The discrete logistic growth equation is given by

$$P_{n+1} = F(P_n) = P_n + rP_n \left(1 - \frac{P_n}{M}\right),$$

where n is in weeks. From the data above, find the best values of the parameters P_0 , M , and r , using the least squares best fit to the data. Write the values of these parameters and the formula for the discrete logistic growth model that best fits the lesser grain borer population in the study above. Also, give the value of the sum of square error between the data and the model with the best fitting parameters. Use Excel to create a graph that shows the data (as points only) and the best fitting model (as straight lines connecting each week of simulated data). Record the simulated populations for weeks 5, 10, 15, 20, 25, and 35.

b. For the discrete logistic growth model found above, determine all equilibria. Use the techniques from class to determine the stability of these equilibria, giving the values of the derivatives at each of the equilibria.

c. The discrete logistic growth equation has problems for large populations because the quadratic function can become negative. Population biologists have used a variety of other updating functions to simulate populations. One discrete model is Ricker's model (usually applied to fish populations), which is given by

$$P_{n+1} = R(P_n) = aP_n e^{-bP_n},$$

where n is in weeks. From the data above, find the best values of the parameters P_0 , a , and b , using the least squares best fit to the data. Write the values of these parameters and the formula for Ricker's growth model that best fits the lesser grain borer population in the study above. Again, give the value of the sum of square error between the data and the model with the best fitting parameters. Which model better fits the data according to the sum of square error? Use Excel to create another graph that shows the data (as points only) and the best fitting Ricker's model (as straight lines connecting each week of simulated data). Include the logistic growth model on this graph to show the comparison. Record the simulated populations from Ricker's model for weeks 5, 10, 15, 20, 25, and 35.

d. For Ricker's growth model, determine all equilibria. Use the techniques from class to determine the stability of these equilibria, giving the values of the derivatives at each of the equilibria.

e. Take the data for *Rhizopertha dominica*, the lesser grain borer, and plot P_{n+1} vs. P_n . To this graph add the identity map and the updating functions $F(P_n)$ (logistic growth function) and $R(P_n)$ (Ricker's growth function). Graph the functions for $0 \leq P_n \leq 1000$ with range $0 \leq P_{n+1} \leq 500$. How do these functions compare, especially near the data? Find the P_n -intercepts for both $F(P_n)$ and $R(P_n)$. Find the maximum for both $F(P_n)$ and $R(P_n)$. Do either of these functions have vertical or horizontal asymptotes? List the asymptotes.