1. A study on a model for breathing the inert gas Argon (Ar) showed that the concentration of Ar in the lungs, c_n after n breaths can be given by the discrete dynamical model

$$c_{n+1} = B(c_n) = (1-q)c_n + q\gamma,$$

where $q = 0.15$ is the fraction of air exchanged and $\gamma = 0.01$ is the atmospheric concentration of Ar.

a. Suppose that initially $c_0 = 0.1$, then find the concentrations c_1 and c_2 of Ar after the first two breaths.

b. Find the equilibrium concentration for this model. Also, determine the derivative of the updating function, $B'(c)$, and use this to determine the behavior of the solution near the equilibrium.

2. Consider the Logistic growth model given by the discrete dynamical model

$$P_{n+1} = F(P_n) = 3.1P_n - 0.0002P_n^2,$$

where P_n is the population after n generations.

a. Suppose that initially there are 1000 individuals, so $P_0 = 1000$. Find the populations at the end of the first three generations P_1, P_2, and P_3.

b. Find the equilibria for this model, then use the derivative of the updating function, $F'(P)$, to determine the behavior of the solution near the equilibrium.

c. Sketch the updating function and the identity function ($P_{n+1} = P_n$), showing the vertex of $F(P)$, the points of intersection, and any intercepts.

3. The San Diego Zoo discovered that because their flamingo population was too small, it would not reproduce until they borrowed some from Sea World. Scientists have discovered that certain gregarious animals require a minimum number of animals in a colony before they reproduce successfully. This is called the *Allee effect*. Consider the following model for the population of a gregarious bird species, where the population, N_n, is given in thousands of birds:

$$N_{n+1} = N_n + 0.2N_n\left(1 - \frac{1}{16}(N_n - 6)^2\right).$$

a. Assume that the initial population is $N_0 = 4$, then determine the population for the next two generations (N_1 and N_2).

b. Find all equilibria for this model.

c. The model above can be expanded to give

$$N_{n+1} = A(N_n) = \frac{3}{4}N_n + \frac{3}{20}N_n^2 - \frac{1}{80}N_n^3.$$

Find the derivative of $A(N)$. Evaluate the derivative $A'(N)$ at each of the equilibria found above and determine the local behavior of the solution near each of those equilibria.

d. Give a brief biological description of what your results imply about this gregarious species of bird.

4. Many biologists in fishery management use Ricker's model to study the population of fish. Let P_n be the population of fish in any year n, then Ricker's model is given by

$$P_{n+1} = R(P_n) = aP_n e^{-bP_n}.$$

Suppose that the best fit to a set of data gives $a = 5$ and $b = 0.004$ for the number of fish sampled from a particular river.

a. Let $P_0 = 100$, then find P_1, P_2, and P_3.

b. Sketch a graph of $R(P)$ with the identity function, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

5. In fishery management, it is important to know how much fishing can be done without severely harming the population of fish. A modification of Ricker's model that includes fishing is given by the model:

$$P_{n+1} = F(P_n) = aP_n e^{-bP_n} - hP_n,$$

where $a = 4$ and $b = 0.002$ are the constants in Ricker's equation that govern the dynamics of the fish population without any fishing and h is the intensity of harvesting fish.

a. Let $h = 0.5$ and $P_0 = 100$, then find P_1, P_2, and P_3.

b. With $h = 0.5$, find all equilibria for this model and describe the behavior of these equilibria.

c. Find all equilibria for this model and describe the behavior of these equilibria when $h = 1$ and $h = 2$.

d. How intense can the fishing be before this population of fish is driven to extinction? That is, find the value of h that makes the only equilibrium be zero (or less than zero).

6. Chalones are proposed chemical agents that cells secret to inhibit cell division by neighboring cells, creating a negative feedback control to maintain a stable number of cells. Some claimed in the 1960s that a breakdown of this control is a cause of cancer. Consider the chalone model for mitosis given by the equation

$$P_{n+1} = f(P_n) = \frac{2P_n}{1 + (bP_n)^c},$$

where $b = 0.05$ and $c = 2$.

a. Let $P_0 = 10$, then find P_1, P_2, and P_3.

b. Sketch a graph of $f(P)$ with the identity function for $P \geq 0$, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.

7. Hassell's model is used to study population of insects. Let P_n be the population of a species of moth in week n and suppose that Hassell's model is given by

$$P_{n+1} = H(P_n) = \frac{aP_n}{(1 + bP_n)^c}.$$

Suppose that the best fit to a set of data gives $a = 5$, $b = 0.002$, and $c = 4$. for this species of beetle.

a. Let $P_0 = 100$, then find P_1, P_2, and P_3.

b. Sketch a graph of $H(P)$ with the identity function for $P \geq 0$, showing the intercepts, all extrema, and any asymptotes.

c. Find all equilibria of the model and describe the behavior of these equilibria.