1. A population of bacteria begins with 1,000,000. The population is growing according to the Malthusian growth equation given by

$$P'(t) = 0.03P(t),$$

where $t$ is in minutes. Find how long it takes for this population to double. Find the population after one hour.

2. A population is growing according to the Malthusian growth equation given by

$$P'(t) = rP(t),$$

where $t$ is in years with $P(0) = 500$. Suppose the population doubles every 7 years. Find the rate constant, then determine the population after 20 years.

3. A radioactive substance with a half-life of 30 days decays according to the differential equation:

$$\frac{dR}{dt} = -kR.$$

Suppose that initially there are 5 mg of the substance ($R(0) = 5$). Find the rate constant $k$, then determine the amount of the substance after 10 days.

4. In the table below, you are to find the solution of the differential equation in the left column from the list in the right column. Verify your choice. (The $c$ is an arbitrary constant.)

| | |
|---|---|
| a. $y' = 1 - y$ | $y(t) = ce^{2t}$ |
| b. $y' = -y$ | $y(t) = ce^{-t} + 1$ |
| c. $y' = 1 - 2t$ | $y(t) = ce^{-2t} + t$ |
| d. $y' = 2y$ | $y(t) = ce^{-t}$ |
| e. $y' = 2ty$ | $y(t) = 1 - ce^{t}$ |
| | $y(t) = ce^{t^2}$ |
| | $y(t) = t - t^2 + c$ |

5. The table below has several second order differential equations in the left column. These have two independent solutions, which are in the right column. Verify your choice. (The $c$ is an arbitrary constant.)

| | |
|---|---|
| a. $y'' + 4y = 0$ | $y(t) = ce^{2t}$ |
| b. $y'' + 2y' + 2y = 0$ | $y(t) = c\cos(2t)$ |
| c. $y'' - 4y = 0$ | $y(t) = ce^{-t}$ |
| d. $y'' - y' - 2y = 0$ | $y(t) = ce^{-t}\cos(t)$ |
| | $y(t) = ce^{-2t}$ |
| | $y(t) = c\sin(2t)$ |
| | $y(t) = ce^{t}$ |
| | $y(t) = ce^{-t}\sin(t)$ |

6. A cylindrical bucket has a hole in the bottom. If $h(t)$ is the height of the water at any time $t$ in hours, then the differential equation describing this leaky bucket is given by the equation:

$$\frac{dh(t)}{dt} = -6\sqrt{h(t)}.$$

If initially, there are 4 inches of water in the bucket ($h(0) = 4$), then the solution is given by

$$h(t) = (2 - 3t)^2.$$

Verify that this is the solution to this differential equation and satisfies the initial condition. Sketch a graph of the solution and determine when the bucket empties.

7. A differential equation that describes logistic growth for some animal population is given by the formula:

$$\frac{dp}{dt} = 0.1p\left(1 - \frac{p}{1000}\right).$$

If the initial population is $p(0) = 100$, then the solution to this *logistic growth model* satisfies

$$p(t) = 1000\left(1 + 9e^{-0.1t}\right)^{-1}.$$

Verify that this is the solution to this logistic growth model and satisfies the initial condition. Sketch a graph of the solution and determine what the population is for very large time, *i.e.*, evaluate

$$\lim_{t\to\infty} p(t).$$