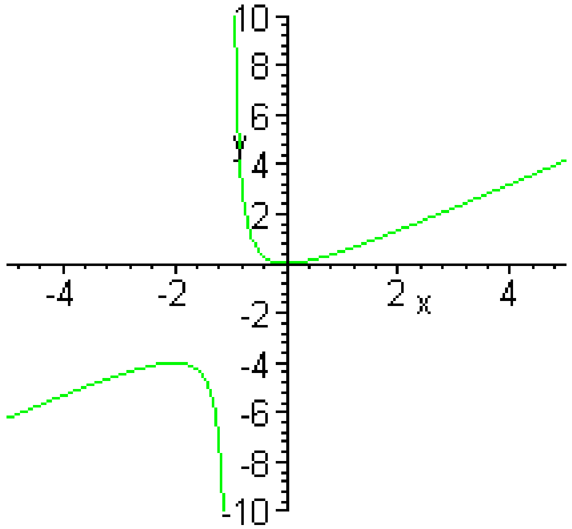
1. $f'(x) = \dfrac{(1-x^2)(3x^2-1/x)+2x(x^3-\ln(x))}{(1-x^2)^2} - 4x^{-3}$,

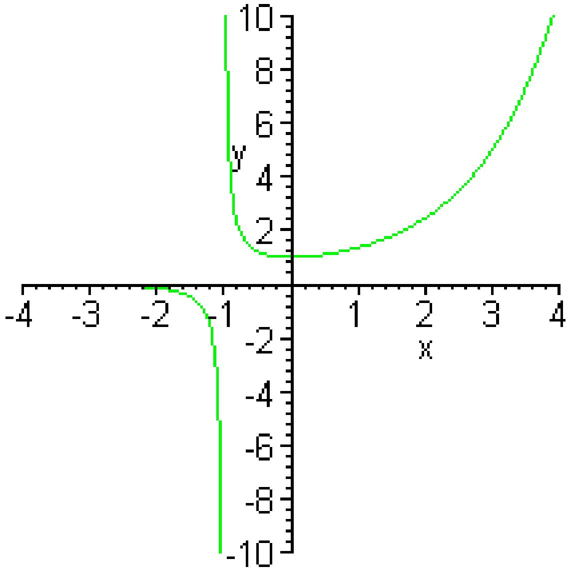
2. $f'(x) = \dfrac{(3x+1)(2x+e^{-x})-3(x^2-e^{-x})}{(3x+1)^2} + (1-x)e^{-x}$,

3. $f'(x) = \dfrac{\frac{1}{2}(2+x)x^{-1/2}-\sqrt{x}}{(2+x)^2} + 3e^{-3x}$,

4. $f'(x) = \dfrac{2x(x^2-e^x)-(x^2+5)(2x-e^x)}{(x^2-e^x)^2} - \dfrac{(2x+1)^2e^{2x}-2xe^{2x}}{(2x+1)^2}$.

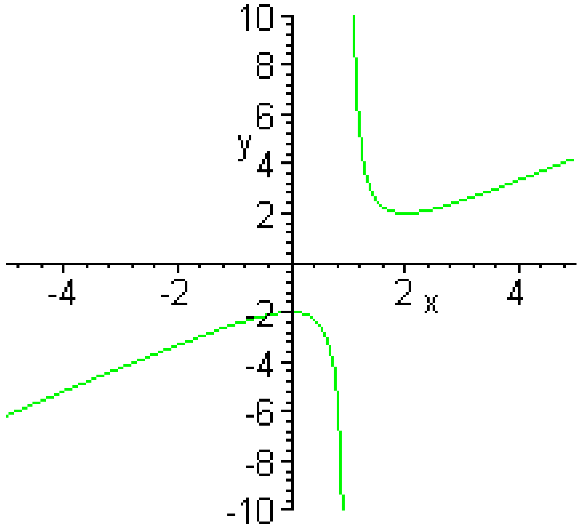
5. $y' = \dfrac{x(x+2)}{(x+1)^2}$. Domain: $x \neq -1$. Maximum at $(-2,-4)$ and minimum at $(0,0)$. Only intercept at $(0,0)$. Vertical asymptote: $x=-1$. Graph is to the left below.

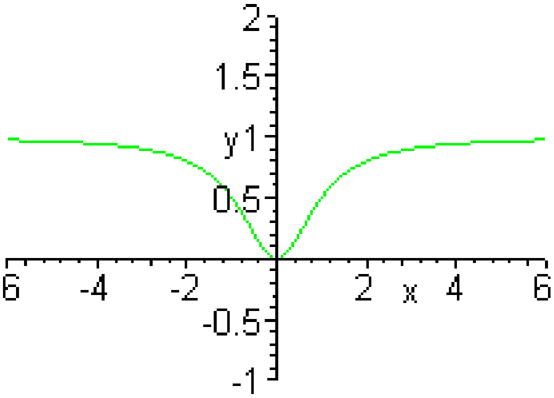




6. $y' = \dfrac{xe^x}{(x+1)^2}$. Domain: $x \neq -1$. Only a y-intercept at $(0,1)$. Vertical asymptote at $x=-1$ and horizontal asymptote at $y=0$ (for $x \to -\infty$). Minimum at $(0,1)$. Graph is to the right above.

7. $y' = \dfrac{x(x-2)}{(x-1)^2}$. Domain: $x \neq 1$. Maximum at $(0,-2)$ and minimum at $(2,2)$. Only a y-intercepts at $(0,-2)$. Vertical asymptote: $x = 1$. Graph is to the left below.



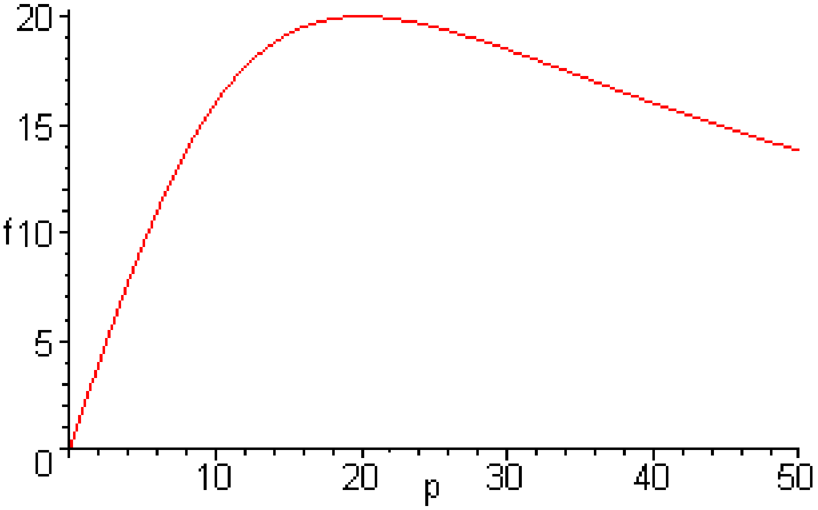


8. $y' = \dfrac{2x}{(x^2+1)^2}$. Domain: All x. Minimum at $(0,0)$. Only intercept is $(0,0)$. Horizontal asymptote: $y = 1$. Graph is to the right above.

9. a. The equilibria are $P_e = 0$ and 20.

b. The derivative is $f'(P_n) = \dfrac{2 - 0.005\,{P_n}^2}{\left(1 + 0.0025\,{P_n}^2\right)^2}$. The maximum mitotic increase occurs at $P_n = 20$, which is also the equilibrium.

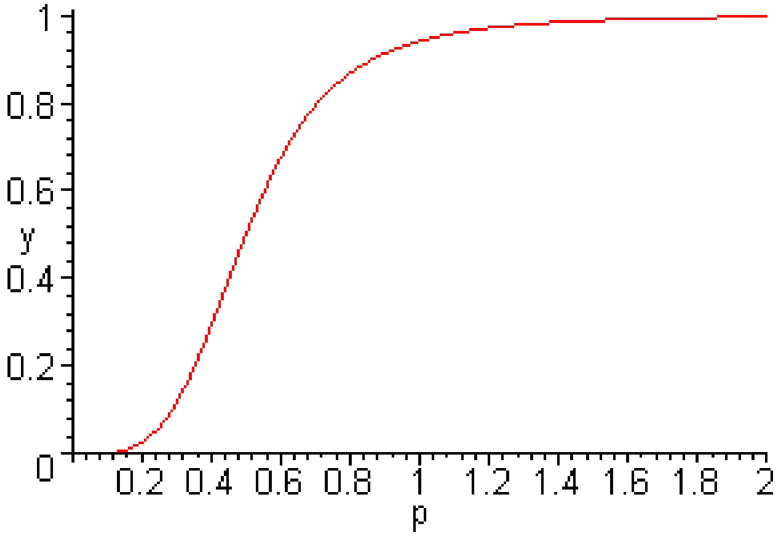
c. A sketch of $f(P)$ is below. The only intercept is $(0,0)$. Horizontal asymptote: $P_{n+1} = 0$. Maximum at $(20,20)$.

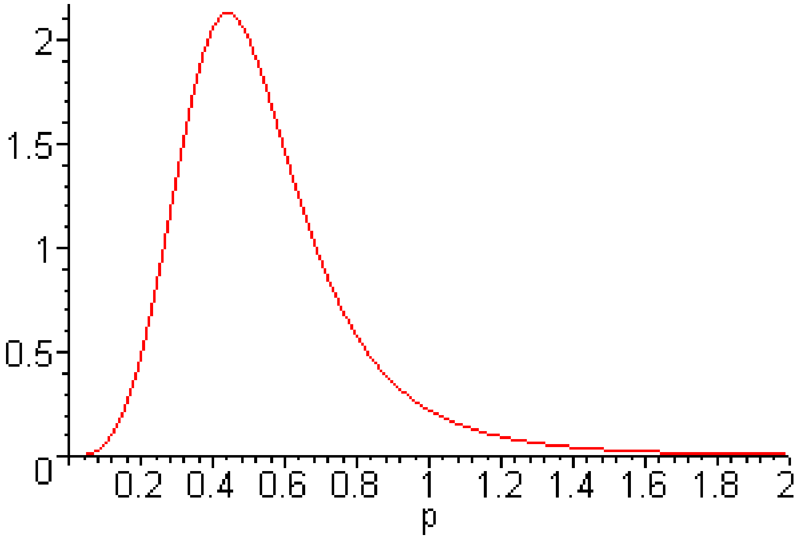


10. a. The derivative is $y'(p) = \dfrac{0.25p^3}{(0.0625+p^4)^2}$. The second derivative is $y''(p) = \dfrac{p^2(0.046875-1.25p^4)}{(0.0625+p^4)^3}$. The second derivative is 0 when $p = 0$ or ± 0.4401. Thus, there are points of inflection at $(0,0)$ and $(0.4401, 0.375)$.

b. Only intercept is $(0,0)$. Horizontal asymptote: $y = 1$. (This is an even function.) A sketch of $y(p)$ is below to the left. The function reaches a 90% saturation when $p = 0.866$. This curve is similar in shape to the O_2 dissociation curve, but the point of inflection occurs at $p = 21.2$ torr, which is about 50 times higher than the point of inflection for the CO dissociation curve,, which implies that hemoglobin binds CO much more strongly than O_2.

c. The only intercept is $(0,0)$. Horizontal asymptote: $y' = 0$. (This is an odd function.) A sketch of $y'(p)$ is below to the right. The maximum for $y'(p)$ is $(0.4401, 2.13)$. Clearly, the p-value of the maximum matches the p-value for the point of inflection.

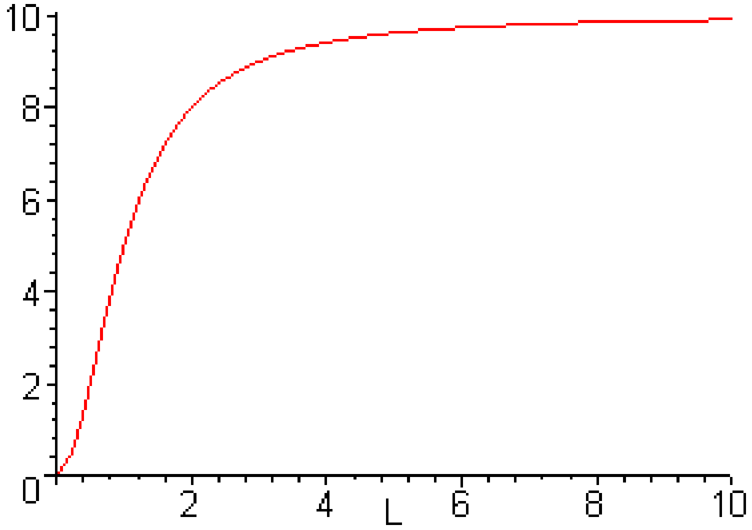


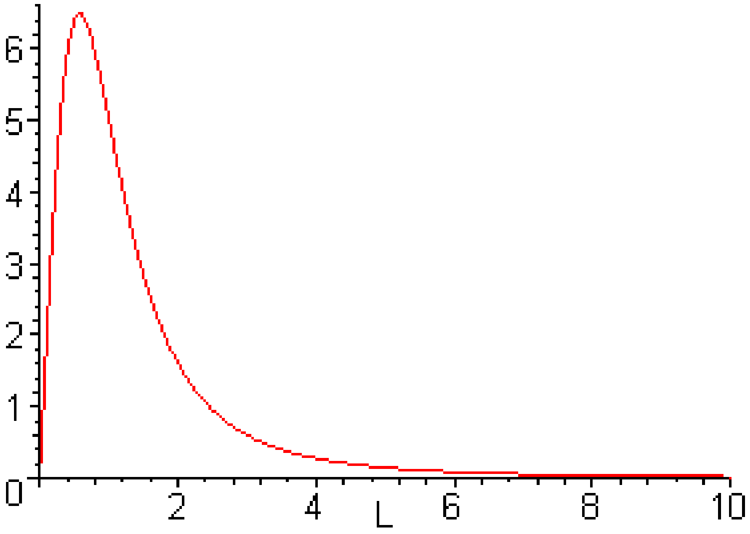


11. a. The derivative is $R'(L) = \dfrac{20L}{(1+L^2)^2}$. The second derivative is $R''(L) = \dfrac{20(1-3L^2)}{(1+L^2)^3}$. The second derivative is 0 when $L = \frac{1}{\sqrt{3}} \simeq 0.5774$. Thus, there is a points of inflection at $(0.5774, 2.5)$.

b. Only intercept is $(0,0)$. Horizontal asymptote: $R = 20$. (This is an even function.) A sketch of $R(L)$ is below to the left.

c. The only intercept is $(0,0)$. Horizontal asymptote: $R' = 0$. (This is an odd function.) A sketch of $R'(L)$ is below to the right. The maximum for $R'(L)$ is $(0.5774, 6.495)$. Clearly, the L-value of the maximum matches the L-value for the point of inflection.

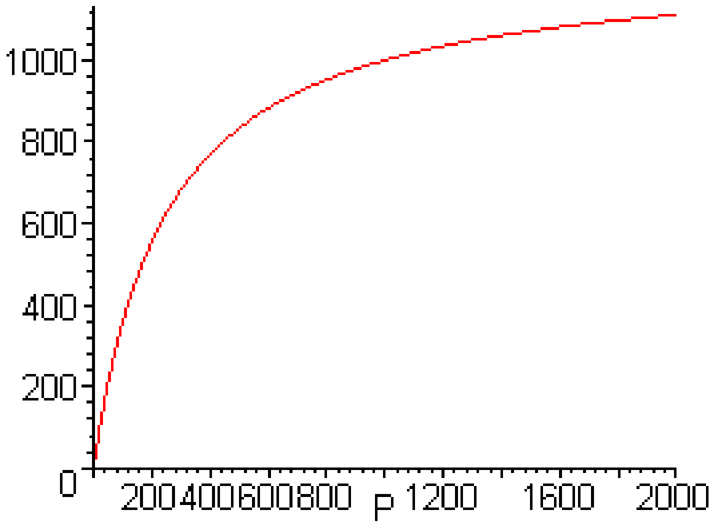


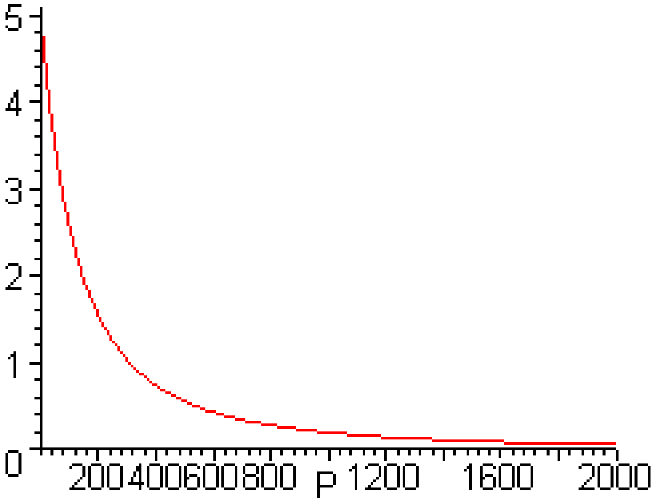


12. a. The derivative is $H'(P) = \dfrac{5}{(1+0.004P)^2}$. The second derivative is $H''(P) = -\dfrac{0.04}{(1+0.004P)^3}$. Since the second derivative is never zero, there are no points of inflection.

b. Only intercept is $(0, 0)$. Horizontal asymptote: $H = 1250$. A sketch of $H(P)$ is below to the left.

c. The only intercept is $(0, 5)$. Horizontal asymptote: $H' = 0$. Since $H'(P)$ is a decreasing value, the maximum increase for $H(P)$ (for $P \geq 0$) occurs at $P = 0$ with $H'(0) = 5$. A sketch of $H'(P)$ is below to the right.





13. a. The derivative is $Y'(t) = \dfrac{1900e^{-0.1t}}{(1+19e^{-0.1t})^2}$. The second derivative is $Y''(t) = \dfrac{190e^{-0.1t}(19e^{-0.1t}-1)}{(1+19e^{-0.1t})^3}$. The second derivative is 0 when $t = 29.44$. Thus, there is a points of inflection at $(29.44, 500)$.

b. Only intercept is $(0, 50)$. Horizontal asymptote: $Y = 1000$. A sketch of $Y(t)$ is below to the left. This population doubles in $t = 7.47$ hr.

c. $Y(t)$ is increasing most rapidly at $t = 29.44$, and is increasing at a rate of 25 yeast/cc/hr. The only intercept is $(0, 4.75)$. Horizontal asymptote: $Y' = 0$. A sketch of $Y'(t)$ is below to the right. The maximum for $Y'(t)$ is $(29.44, 25)$.

d. The doubling time for the Malthusian growth model is $t = 6.93$ hr.