3. We rewrite $f(x) = \dfrac{\sqrt{x}}{2+x} - \dfrac{1}{e^{3x}} = \dfrac{x^{1/2}}{2+x} - e^{-3x}$. Applying the quotient rule to the first and standard differentiation of exponentials to the second, we have:

$$f'(x) = \frac{(2+x)\frac{1}{2}x^{-1/2} - x^{1/2}(1)}{(2+x)^2} + 3e^{-3x}.$$

4. For $f(x) = \dfrac{x^2+5}{x^2-e^x} - \dfrac{xe^{2x}}{2x+1}$, we apply the quotient rule twice using the product rule in the numerator of the second expression.

$$f'(x) = \frac{2x(x^2-e^x) - (x^2+5)(2x-e^x)}{(x^2-e^x)^2} - \frac{(2x+1)(2xe^{2x}+e^{2x}) - 2xe^{2x}}{(2x+1)^2}.$$

6. $y = \dfrac{e^x}{x+1}$,

Derivative: By the quotient rule, $y' = \dfrac{(x+1)e^x - e^x(1)}{(x+1)^2} = \dfrac{xe^x}{(x+1)^2}$.

Domain: $x \neq -1$.

y-intercept: $y(0) = 1$, so $(0, 1)$.

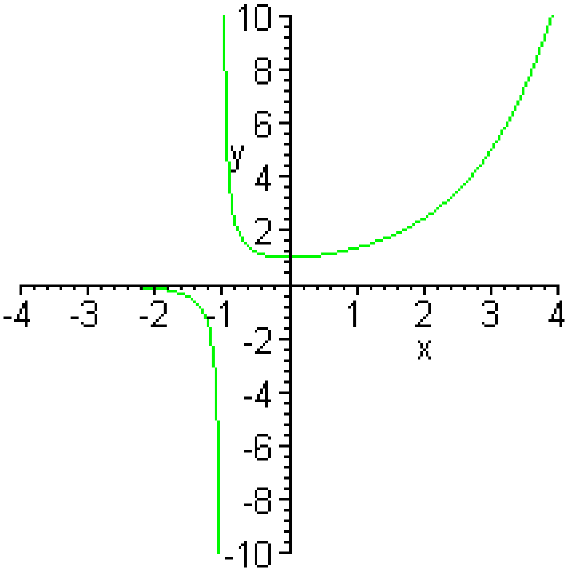
x-intercept: Since the exponential function is not zero, there are no x-intercepts.

Horizontal asymptote: As $x \to -\infty$, $y \to 0$, so $y = 0$ is a horizontal asymptote (looking to the left).

Vertical asymptote: Since the denominator $x + 1 = 0$ at $x = -1$, this is a vertical asymptote.

Critical points satisfy $y'(x) = 0$, so $xe^x = 0$ or $x = 0$. With $y(0) = 1$, $(0, 1)$ is a minimum.

The graph is below.



7. $y = \dfrac{x^2 - 2x + 2}{x - 1}$,

Derivative: By the quotient rule, $y' = \dfrac{(x-1)(2x-2) - (x^2 - 2x + 2)(1)}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2}$.

Domain: $x \neq 1$.

y-intercept: $y(0) = -2$, so $(0, -2)$.

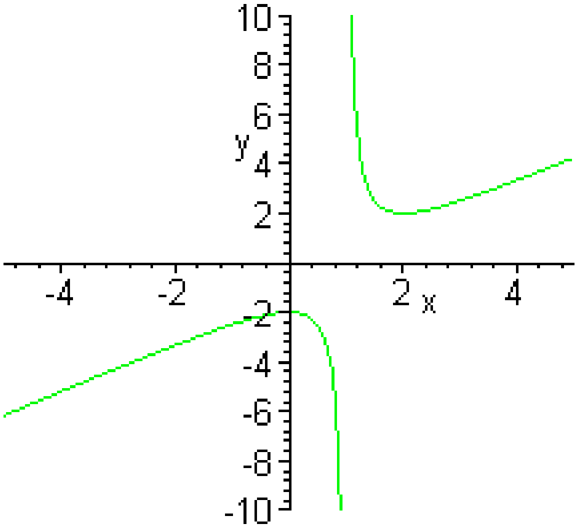
x-intercept: Since $x^2 - 2x + 2 = 0$ has no real solutions, there are no x-intercepts.

There are no horizontal asymptotes.

Vertical asymptote: Since the denominator $x - 1 = 0$ at $x = 1$, this is a vertical asymptote.

Critical points satisfy $y'(x) = 0$, so $x(x-2) = 0$ or $x = 0$ and $x = 2$. With $y(0) = -2$, $(0, -2)$ is a maximum. (We see that $y' > 0$ for $x < 0$ and $y' < 0$ for $x > 0$, giving the maximum.) Similarly, since $y(2) = 2$, $(2, 2)$ is a minimum.

The graph is below.



9. a. The equilibria satisfy

$$P_e = \frac{2p_e}{1+0.0025P_e^2} \quad \text{or} \quad P_e(1+0.0025P_e^2) = 2P_e.$$

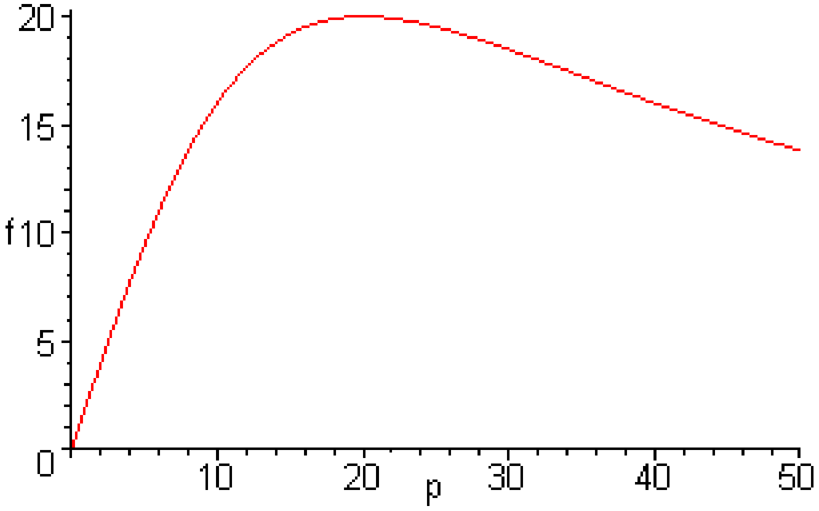
Thus, either $P_e = 0$ or $1 + 0.0025P_e^2 = 2$. The latter implies that $0.0025P_e^2 = 1$ or $P_e^2 = 400$. Thus, $P_e = \pm 20$, but since the population density cannot be negative $P_e = 20$.

b. From the quotient rule,

$$\begin{aligned} f'(P_n) &= \frac{(1+0.0025P_n^2)2 - 2P_n(0.005P_n)}{\left(1+0.0025\,{P_n}^2\right)^2} \\ &= \frac{2-0.005\,{P_n}^2}{\left(1+0.0025\,{P_n}^2\right)^2}. \end{aligned}$$

The maximum occurs when $f'(P_n) = 0$, which is when the numerator above is zero. Thus, $2 - 0.005P_n^2 = 0$ or $P_n^2 = 400$. It follows that the maximum mitotic increase occurs at $P_n = 20$, which is also the equilibrium.

c. A sketch of $f(P)$ is below. The only intercept is $(0,0)$. As $P_n \to \infty$, the denominator of $f(P_n)$ gets larger faster than the numerator (higher power of P_n), so $f(P_n) \to 0$, so there is a horizontal asymptote at $P_{n+1} = 0$. From Part b., the maximum occurs at $(20,20)$.



11. a. By the quotient rule, the derivative is

$$\begin{aligned} R'(L) &= \frac{20L(1+L^2)-10L^2(2L)}{(1+L^2)^2} \\ &= \frac{20L}{(1+L^2)^2} = \frac{20L}{1+2L^2+L^4}. \end{aligned}$$

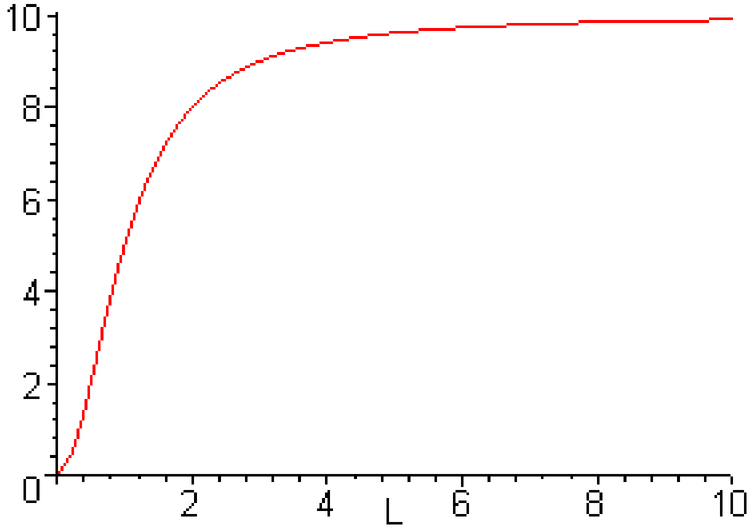
The second derivative is

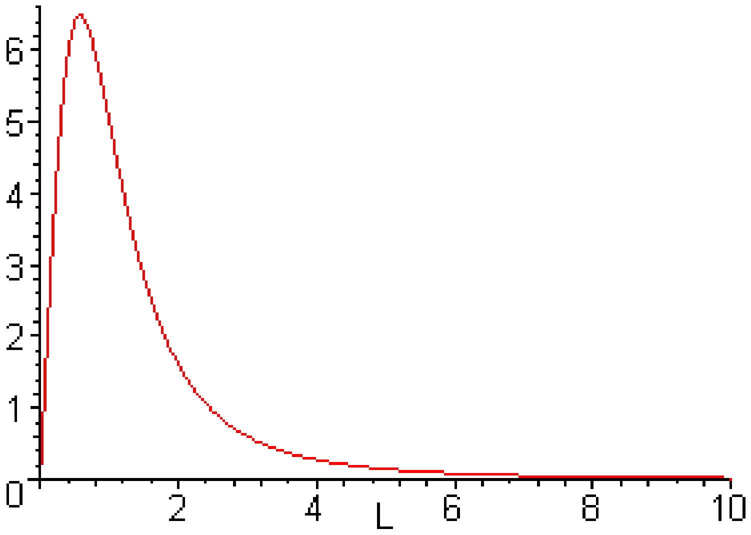
$$\begin{aligned} R''(L) &= \frac{20(1+2L^2+L^4)-20L(4L+4L^3)}{(1+L^2)^4} \\ &= \frac{20(1-3L^2)}{(1+L^2)^3}. \end{aligned}$$

The second derivative is 0 when the numerator of the above expression is zero. Thus, $1-3L^2=0$ or $L=\frac{1}{\sqrt{3}} \simeq 0.5774$. (Note we take only the positive square root because of the domain.) Thus, there is a points of inflection at $(0.5774, 2.5)$.

b. Only intercept is $(0,0)$. Since the power of the numerator is equal to the power of the denominaor, there is a horizontal asymptote. Considering only the highest power of L, we see that the horizontal asymptote is $R=20$. A sketch of $R(L)$ is below to the left.

c. The only intercept is $(0,0)$. For the derivative, the power of the denominator is greater than the power of L in the numerator, so the horizontal asymptote is $R'=0$. (This is an odd function.) A sketch of $R'(L)$ is below to the right. Since the second derivative is zero at $L=0.5774$, there is a maximum for $R'(L)$ at $(0.5774, 6.495)$. Clearly, the L-value of the maximum matches the L-value for the point of inflection.





13. a. By the quotient rule, the derivative is

$$\begin{aligned} Y'(t) &= \frac{(1+19e^{-0.1t})0 - 1000((-1.9)e^{-0.1t})}{(1+19e^{-0.1t})^2} \\ &= \frac{1900e^{-0.1t}}{(1+19e^{-0.1t})^2} = \frac{1900e^{-0.1t}}{1+38e^{-0.1t}+361e^{-0.2t}}. \end{aligned}$$

The second derivative is

$$\begin{aligned} Y''(t) &= \frac{-190e^{-0.1t}(1+38e^{-0.1t}+361e^{-0.2t}) - 1900e^{-0.1t}(-3.8e^{-0.1t}-72.2e^{-0.2t})}{(1+19e^{-0.1t})^4} \\ &= \frac{190e^{-0.1t}(19e^{-0.1t}-1)}{(1+19e^{-0.1t})^3}. \end{aligned}$$

The second derivative is 0 when $19e^{-0.1t} - 1 = 0$ or $e^{0.1t} = 19$. $t = 10\ln(19) = 29.44$. Thus, there is a point of inflection at $(29.44, 500)$.

b. Only intercept is $(0, 50)$. As $t \to \infty$, $e^{-0.1t} \to 0$, so $Y(t) \to 1000$, which gives a horizontal asymptote of $Y = 1000$. A sketch of $Y(t)$ is below to the left. Since the population starts at 50, it doubles when it reaches 100. Solving $Y(t) = \frac{1000}{1+19e^{-0.1t}} = 100$ gives $1 + 19e^{-0.1t} = 10$, so $e^{0.1t} = \frac{19}{9}$. Thus, this population doubles when $t = 10\ln\left(\frac{19}{9}\right) = 7.47$ hr.

c. $Y(t)$ is increasing most rapidly at the point of inflection, so $t = 29.44$ hr. Substituting this value into the derivative gives the population increasing at a rate of 25 yeast/cc/hr. The only intercept is $(0, 4.75)$. Since the numerator has a decaying exponential function, the horizontal asymptote is $Y' = 0$. A sketch of $Y'(t)$ is below to the right. The maximum for $Y'(t)$ is $(29.44, 25)$.

d. The Malthusian growth model doubles when it reaches 100. Solving $100 = 50e^{0.1t}$ gives $e^{0.1t} = 2$ or $t = 10\ln(2)$. Thus, the doubling time for the Malthusian growth model is $t = 6.93$ hr.