1. a. For $f(t) = 2t^2 + t$ and $g(t) = t - 2$,

$$\begin{aligned} f(0) &= 2(0)^2 + 0 = 0, \\ f(2) &= 2(2)^2 + 2 = 10, \\ g(-2) &= -2 - 2 = -4, \\ g(3) &= 3 - 2 = 1. \end{aligned}$$

b. The composite functions $f(g(t))$ and $g(f(t))$ satisfy

$$\begin{aligned} f(g(t)) &= 2(g(t))^2 + g(t) = 2(t-2)^2 + (t-2) = 2t^2 - 8t + 8 + t - 2 = 2t^2 - 7t + 6 \\ g(f(t)) &= f(t) - 2 = 2t^2 + t - 2 \end{aligned}$$

c. $f(g(1)) = 2(1)^2 - 7(1) + 6 = 1$ and $g(f(1)) = 2(1)^2 + 1 - 2 = 1$.

2. a. The function $f(x) = 30 + x - x^2$ is a parabola opening down. The vertex occurs at $x = \frac{1}{2}$, so $f(1/2) = 30 + \frac{1}{2} - \left(\frac{1}{2}\right)^2 = 30.25$. Therefore, the range is $-\infty < x < 30.25$.

b. If $f(x) > 0$, then $30 + x - x^2 = (6 - x)(5 + x) > 0$, so the domain is $-5 < x < 6$.

4. $x^2 + 4x - 3 = 0$, so the quadratic formula gives $x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-3)}}{2} = \frac{-4 \pm \sqrt{16+12}}{2} = -2 \pm \sqrt{7}$.

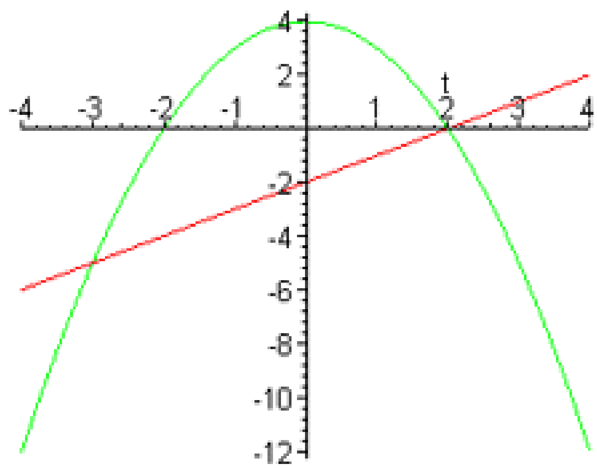
6. $x^2 - x - 20 = (x - 5)(x + 4) = 0$, so $x = 5, -4$.

8. $x^2 - 9 = (x + 3)(x - 3) = 0$, so $x = \pm 3$.

10. $x^2 - 2x + 2 = 0$, so the quadratic formula gives $x = \frac{2 \pm \sqrt{4 - 4(1)2}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$. This is a complex solution, hence there is no real solution.

11. For $f(t) = 4 - t^2$, the t-intercepts satisfy $t^2 - 4 = 0$, so $t = \pm 2$. The y-intercept satisfies $f(0) = 4$. The vertex occurs at $x = 0$, so the vertex is the y-intercept, $(0, 4)$. For the line, $g(t) = t - 2$, the t-intercept satisfies $t - 2 = 0$ or $t = 2$. The y-intercept occurs at $(0, -2)$. The slope is $m = 1$.

The curves intersect when $f(x) = g(x)$ or $4 - t^2 = t - 2$. Thus, $t^2 + t - 6 = (t+3)(t-2) = 0$, which implies that $t = 2$ and -3. Since $g(-3) = f(-3) = -5$ and $g(2) = f(2) = 0$, so the points of intersection are $(-3, -5)$ and $(2, 0)$.



14. From the notes,

$$[\mathrm{H}^+] = \frac{1}{2}\left(-K_a + \sqrt{K_a^2 + 4K_a x}\right),$$

where x is the normality of the solution. For a 0.1N solution, $x = 0.1$, so

$$[\mathrm{H}^+] = \frac{1}{2}\left(-1.75 \times 10^{-5} + \sqrt{(1.75 \times 10^{-5})^2 + 0.4(1.75 \times 10^{-5})}\right) = 0.001314.$$

It follows that the pH $= -\log_{10} 0.001314 = 2.881$.

Similarly, for a 1N solution, $x = 1$, so

$$[\mathrm{H}^+] = \frac{1}{2}\left(-1.75 \times 10^{-5} + \sqrt{(1.75 \times 10^{-5})^2 + 4(1.75 \times 10^{-5})}\right) = 0.004175.$$

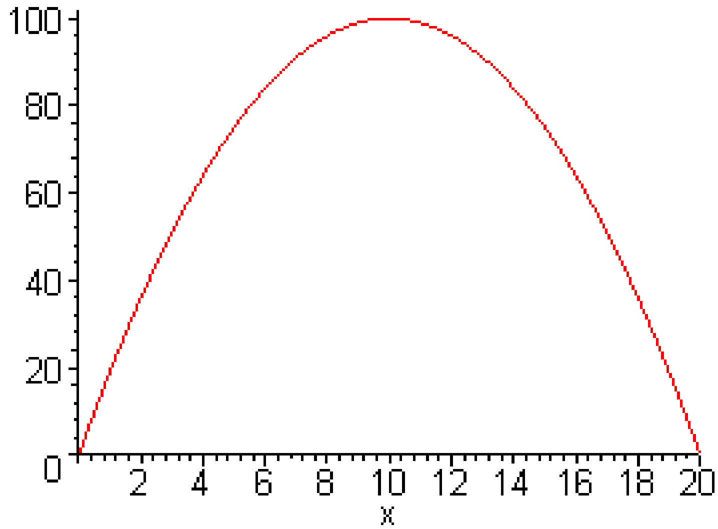
It follows that the pH $= -\log_{10} 0.004175 = 2.379$.

15. a. The perimeter of the rectangle is 40 cm, so $2y + 2x = 40$. It follows that $y = 20 - x$.

b. The area of a rectangle is $A = xy$, so

$$A(x) = 20x - x^2.$$

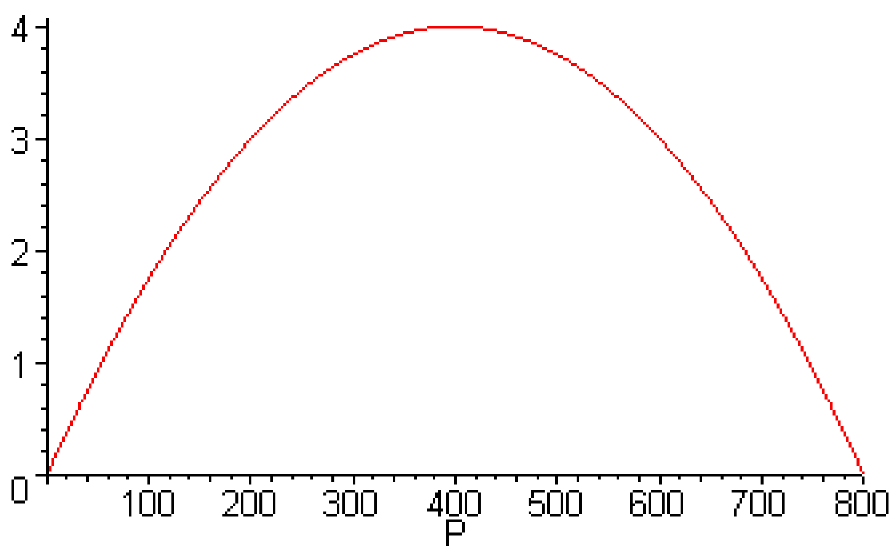
c. $A(x)$ is a parabola. The domain of $A(x)$ is $0 < x < 20$. The vertex occurs at $x = 10$ with $A(10) = 100$ cm^2. Thus, the particular rectangle is a square. The graph is shown below.



17. a. A sketch of the graph $g(P) = 0.02P - 0.000025P^2$ is shown below.

b. The equilibrium population satisfies $0 = 0.02P_e - 0.000025P_e^2 = 0.02P_e(1 - 0.000125P_e)$, so $P_e = 0$ or $P_e = \frac{1}{0.000125} = 800$ individuals.

c. The growth rate occurs at the vertex of parabola, which satisfies $P = 400$, the midpoint between the P-intercepts (or equilibria) of the growth function. Thus, the maximum growth rate is $g(400) = 0.02(400) - 0.000025(400)^2 = 4$ individuals per generation.



19. a. From the lectures,

$$\begin{aligned} J(k) &= e_1^2 + e_2^2 + e_3^2 = (20 - 3.3k)^2 + (2 - 0.5k)^2 + (12 - 2k)^2 \\ &= 400 - 2(20)(3.3)k + (3.3k)^2 + 4 - 2(2)(0.5)k + (0.5k)^2 + 144 - 2(12)(2)k + (2k)^2 \\ &= 15.14k^2 - 182k + 548. \end{aligned}$$

The k-value of the vertex is given by $k = 182/(2(15.14)) = 6.01$. From this, we substitute into the sum of squares function $J(k)$ to obtain the least sum of squares

$$J(6.01) = 15.14(6.01)^2 - 182(6.01) + 548 = 1.038.$$

b. Using the best slope in the model, we see that her great-grandfather has a height of $d = 6.01(1) = 6.01$ ft, so her mother has the better memory.