1. a. Since $P_0 = 50,000$ and $r = 0.08$, the general solution to this Discrete Malthusian growth model is given by

$$P_n = 50,000(1.08)^n.$$

It follows that

$$\begin{aligned} P_1 &= 50,000(1.08) = 54,000 \\ P_2 &= 50,000(1.08)^2 = 58,320 \\ P_3 &= 50,000(1.08)^3 = 62,986. \end{aligned}$$

The amount of time required for this population to double is found by solving

$$\begin{aligned} 50,000(1.08)^n &= 100,000 \\ (1.08)^n &= 2 \\ n\ln(1.08) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.08)} = 9.01 \text{ hr} \end{aligned}$$

b. With $P_0 = 250,000$ and $r = 0.06$, the general solution to this Discrete Malthusian growth model is given by

$$P_n = 250,000(1.06)^n.$$

It follows that $P_1 = 250,000(1.06) = 265,000$, $P_2 = 250,000(1.06)^2 = 280,000$, and $P_3 = 250,000(1.06)^3 = 297,754$. The amount of time required for this population to double is found by solving

$$\begin{aligned} 250,000(1.06)^n &= 500,000 \\ (1.06)^n &= 2 \\ n\ln(1.06) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.06)} = 11.9 \text{ hr.} \end{aligned}$$

3. a. Since the population of the U. S. in 1980 was 227 million and in 1990 was 249 million, then

$$\frac{P_1}{P_0} = \frac{249}{227} = 1.09692 = 1 + r.$$

thus, the growth constant is $r = 0.09692$. It follows that the general solution is given by

$$P_n = 227(1.09692)^n,$$

where n is in decades after 1980. The population in 2000 satisfies

$$P_2 = 227(1.09692)^2 = 273.1 \text{ million},$$

while the population in 2020 is given by

$$P_4 = 227(1.09692)^4 = 328.6 \text{ million}.$$

b. Since the population of Mexico in 1980 was 69 million and in 1990 was 85 million, then

$$\frac{P_1}{P_0} = \frac{85}{69} = 1.2319 = 1 + r.$$

Thus, the growth constant is $r = 0.2319$. It follows that the general solution is given by

$$P_n = 69(1.2319)^n,$$

where n is in decades after 1980. The population in 2000 satisfies

$$P_2 = 69(1.2319)^2 = 104.7 \text{ million},$$

while the population in 2020 is given by

$$P_4 = 69(1.2319)^4 = 158.9 \text{ million}.$$

The amount of time required for this population to double is found by solving

$$\begin{aligned} 69(1.2319)^n &= 138 \\ (1.2319)^n &= 2 \\ n\ln(1.2319) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.2319)} = 3.32 \text{ decades} = 33.2 \text{ years}. \end{aligned}$$

c. The two populations are equal when

$$\begin{aligned} 227(1.09692)^n &= 69(1.2319)^n \\ \left(\frac{1.2319}{1.09692}\right)^n &= \frac{227}{69} \\ n\ln(1.12305) &= \ln(3.290) \\ n &= \frac{\ln(3.290)}{\ln(1.12305)} = 10.26 \text{ decades} = 102.6 \text{ years}. \end{aligned}$$

Thus, the population of Mexico will first exceed that of U. S. in 103 years from 1980 with Mexico having a population of 591.2 million and U. S. having a population of 588.6 million.

4. a. Since the population of the U. S. in 1880 was 50.2 million and in 1890 was 62.9 million, then

$$\frac{P_1}{P_0} = \frac{62.9}{50.2} = 1.2530 = 1 + r.$$

Thus, the growth constant is $r = 0.2530$. It follows that the general solution is given by

$$P_n = 50.2(1.2530)^n,$$

where n is in decades after 1880.

b. From the general formula, we have the population in 1900 is

$$P_2 = 50.2(1.2530)^2 = 78.8 \text{ million}.$$

Since the actual population is only 76.0 million, the percent error satisfies

$$100\left|\frac{78.8-76.0}{76.0}\right| = 3.7.$$

So the model predicts a population of 78.8 million in 1900, which has an error of 3.7% from the actual census data.

c. The amount of time required for this population to double is found by solving

$$\begin{aligned}
50.2(1.2530)^n &= 100.4 \\
(1.2530)^n &= 2 \\
n\ln(1.2530) &= \ln(2) \\
n &= \frac{\ln(2)}{\ln(1.2530)} = 3.07 \text{ decades} = 30.7 \text{ years.}
\end{aligned}$$

6. a. The general solution to this Malthusian growth problem is given by

$$P_n = 5000(1.015)^n.$$

The time required for this population to double is found by solving

$$\begin{aligned}
5000(1.015)^n &= 10000 \\
(1.015)^n &= 2 \\
n\ln(1.015) &= \ln(2) \\
n &= \frac{\ln(2)}{\ln(1.015)} = 46.6 \text{ min.}
\end{aligned}$$

b. If the culture starts with 1000 bacteria, then we can write the general solution, $P_n = 1000(1+r)^n$. If it doubles in 40 min, then

$$\begin{aligned}
1000(1+r)^{40} &= 2000 \\
(1+r)^{40} &= 2 \\
1+r &= (2)^{\frac{1}{40}} = 1.01748
\end{aligned}$$

Thus, the solution is given by $P_n = 1000(1.01748)^n$.

The two populations are equal if

$$\begin{aligned}
5000(1.015)^n &= 1000(1.01748)^n \\
\left(\frac{1.01748}{1.015}\right)^n &= \frac{5000}{1000} = 5 \\
n\ln(1.002443) &= \ln(5) \\
n &= \frac{\ln(5)}{\ln(1.002443)} = 659.6 \text{ min.}
\end{aligned}$$

The level of bacteria in each of the cultures at this time is

$$5000(1.015)^{659.6} = 92,021,937 \text{ bacteria.}$$

7. The general solution for compounded interest with an annual interest rate $r = 6\%$ and an initial investment of $P_0 = \$10{,}000$ is given by the formula

$$P_n = 10000\left(1 + \frac{0.06}{k}\right)^{kn},$$

where k is the number of times the interest is compounded each year and n is the number of years of the investment.

With annual compounding ($k = 1$), the investment for 2 years gives

$$P_2 = 10000(1.06)^2 = \$11,236.00.$$

After 5 years, the investment gives

$$P_5 = 10000(1.06)^5 = \$13,382.26.$$

With semi-annual compounding ($k = 2$), the investment for 2 years gives

$$P_2 = 10000(1.03)^4 = \$11,255.09.$$

After 5 years, the investment gives

$$P_5 = 10000(1.03)^{10} = \$13,439.16.$$

With quarterly compounding ($k = 4$), the investment for 2 years gives

$$P_2 = 10000(1.015)^8 = \$11,264.93.$$

After 5 years, the investment gives

$$P_5 = 10000(1.015)^{20} = \$13,468.55.$$

With monthly compounding ($k = 12$), the investment for 2 years gives

$$P_2 = 10000(1.005)^{24} = \$11,271.60.$$

After 5 years, the investment gives

$$P_5 = 10000(1.005)^{60} = \$13,488.50.$$

9. a. The general solution for the population of herbivores satisfies

$$y_n = 2000(1.05)^n.$$

It follows that $y_1 = 2000(1.05)^1 = 2100$, $y_2 = 2000(1.05)^2 = 2205$, $y_3 = 2000(1.05)^3 = 2315$.

b. The general solution for the competing population of herbivores satisfies

$$y_n = 500(1.07)^n.$$

The time required for this population to double is found by solving

$$\begin{aligned} 500(1.07)^n &= 1000 \\ (1.07)^n &= 2 \\ n\ln(1.07) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.07)} = 10.24. \end{aligned}$$

c. The two populations are equal when

$$\begin{aligned} 2000(1.05)^n &= 500(1.07)^n \\ \left(\frac{1.07}{1.05}\right)^n &= \frac{2000}{500} = 4 \\ n\ln(1.01905) &= \ln(4) \\ n &= \frac{\ln(4)}{\ln(1.01905)} = 73.47. \end{aligned}$$

11. a. The populations for the first 5 days are

day	P_n	$1+k(t)$	P_{n+1}
0	40,000	1.08	43,200
1	43,200	1.07	46,224
2	46,224	1.06	48,997
3	48,997	1.05	51,447
4	51,447	1.04	53,505
5	53,505		

b. The growth rate is zero when $k(t) = 0 = 0.08 - 0.01t$, so $t = 8$ days.

day	P_n	$1+k(t)$	P_{n+1}
5	53,505	1.03	55,110
6	55,110	1.02	56,213
7	56,213	1.01	56,775
8	56,775	1.00	56,775

c. There are two ways to find when the invertebrate goes extinct. First, one iterates until the population drops below one individual in which case the answer is either $t = 52$ days (below 1) or $t = 53$ days (below 0.5). Second, determine when the factor $1 + k(t_n) = 0$, which occurs at $t = 108$ days. The first represents numerical extinction, while the second represents a theoretical upper bound for the extinction.

12. a. The Malthusian growth model for estimating Italy's population is given by

$$P_n = 50.2(1.061)^n.$$

It follows that the population in 1990 is $P_3 = 50.2(1.061)^3 = 60.0$ million, while in 2000 is $P_4 = 50.2(1.061)^4 = 63.6$ million. The amount of time required for the population to double is found by solving

$$\begin{aligned} 50.2(1.061)^n &= 100.4 \\ (1.061)^n &= 2 \\ n\ln(1.061) &= \ln(2) \\ n &= \frac{\ln(2)}{\ln(1.061)} = 11.7 \text{ decades} = 117 \text{ years.} \end{aligned}$$

b. The Nonautonomous Malthusian growth model uses $k(t_n) = 3.598 - 0.0018t_n$ with $t_n = 1960 + 10n$. The table below gives the solution to the Nonautomous Malthusian growth model. The first column is the date. The second column is the population for that date. The third column gives the growth factor $k(t_n)$. The fourth column is equal to the second column times the third and becomes the population prediction for Italy's population in the next decade.

Year	P_n	$1 + k(t_n)$	P_{n+1}
1960	50.2	1.07	53.7
1970	53.7	1.052	56.5
1980	56.5	1.034	58.4
1990	58.4	1.016	59.4
2000	59.4	0.998	

The growth term $k(t) = 3.598 - 0.0018t = 0$ when $t = \frac{3.598}{0.0018} \simeq 1999$, so the population would level off then.

c. The percent error in 1990 is given by

$$100\left|\frac{58.4 - 56.8}{56.8}\right| = 2.9,$$

while the percent error in 2000 is

$$100\left|\frac{59.4 - 57.9}{57.9}\right| = 2.5.$$

Both predicted values are high with the errors being 2.9% in 1990 and 2.5% in 2000. The census data are clearly leveling off now, which is consistent with the predicted leveling off in 1999.