1. Let P_n be the population of some organism after n hours. Suppose that the organism satisfies the Malthusian growth model

$$P_{n+1} = (1 + r)P_n$$

with a growth rate r and an initial population P_0.

a. Let $P_0 = 50,000$ and $r = 0.08$. Find the population of the organism at the end of each of the first 3 hours, *i.e.*, find P_1, P_2, and P_3. Also, determine the amount of time required for this population to double.

b. Repeat the process in Part a for $P_0 = 250,000$ and $r = 0.06$.

2. The population of China in 1980 was about 985 million, and a census in 1990 showed that the population had grown to 1,137 million. Assume that its population is growing according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1980 and P_n is population n decades after 1980.

a. Use the data above to find the growth constant r and then write the general solution P_n. Predict the population in the year 2000.

b. How long does it take for China's population to double?

3. a. The population of the U. S. in 1980 was about 227 million, and a census in 1990 showed that the population had grown to 249 million. Assume that this population grows according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1980, and P_n is population n decades after 1980. Use the data above to find the growth constant r, then write the general solution P_n. Predict the population in the years 2000 and 2020.

b. In 1980, the population of Mexico was 69 million, while in 1990, it had grown to 85 million. Assume its population is also growing according to a Malthusian growth law. Find its rate of growth per decade and predict its population in 2000 and 2020. How long does it take for Mexico's population to double?

c. If these countries continue to grow according to these Malthusian growth laws, then determine the first year when Mexico's population will exceed that of the U. S. and determine their populations at that time.

4. The population of the United States was about 50.2 million in 1880 and 62.9 million in 1890. Let 1880 be represented by P_0 and assume that its population is growing according to the Malthusian growth law,

$$P_{n+1} = (1 + r)P_n,$$

where n is in years.

a. Use the data above to find the annual growth rate r, then write an expression for the population in any year following 1880. (Write the solution P_n in terms of P_0 with n being the number of years after 1880.)

b. Predict the population in the year 1900. The actual population was about 76.0 million. What is the error between the model and the actual census data?

c. According to the model, how long until the U. S. population doubled from its 1880 level?

5. Take $r = 0.15$ and $P_0 = 75,994,575$ (the population of the U. S. in 1900). Use the Malthusian growth model

$$P_{n+1} = (1 + r)P_n,$$

where n is the number of decades after 1900 and P_n is population n decades after 1900. Simulate this model for $n = 1, 2, 3, ..8, 9$ to estimate the population through the 20^{th} century. Compare your results to the actual census data by computing the error at each decade. Also, determine how long the model predicts for the population to double and compare this to the actual data.

6. a. A culture of bacteria satisfies the Malthusian growth equation

$$P_{n+1} = 1.015P_n, \qquad P_n = 5000,$$

where n is in minutes. Solve this growth equation and determine how long it takes for this culture to double.

b. Another culture of bacteria satisfies a similar Malthusian growth law. Suppose that this culture doubles in 40 min and starts with 1000 bacteria. Find the general solution for this culture and determine how long until the population of this bacteria is the same as the original culture from Part a.

7. Consider an annual interest rate $r = 6\%$ and an initial investment of $P_0 = \$10,000$. Find the value of the investment after two years with interest compounded annually, semiannually, quarterly, and monthly. What are the values of the investments after five years?

8. a. A population of bacteria satisfies the growth equation

$$b_{n+1} = rb_n,$$

where $r = 1.05$. If the initial population is $b_0 = 10^6$, then determine the populations b_1, b_2, and b_3. Also, give an expression for the population b_n.

b. Another group of bacteria satisfies the same growth equation, except $r = 1.1$ and $b_0 = 2 \times 10^5$. How long does it take for this population to double?

c. Find when the two populations are equal.

9. a. A population of herbivores satisfies the growth equation $y_{n+1} = 1.05y_n$. If the initial population is $y_0 = 2000$, then determine the populations y_1, y_2, and y_3. Also, give an expression for the population y_n.

b. A competing group of herbivores satisfies the growth equation $z_{n+1} = 1.07z_n$. If the initial population is $z_0 = 500$, then determine how long it takes for this population to double.

c. Find when the two populations are equal.

10. a. You have $10,000 to invest. A Municipal Bond offers an annual interest of 8.25%. The other alternative that you are considering is Treasury Note that gives an annual interest of 8%, but has its interest compounded quarterly. Which of these is the better investment and by how much at the end of the first year?

b. Put your money in the best investment and determine how much money you have after 5 years.

11. An invertebrate living in a pond is effected by a pollutant that is slowly seeping into the ecosystem. The population dynamics for this invertebrate is given by the nonautonomous Malthusian growth model

$$P_{n+1} = (1 + k(t_n))P_n \quad \text{with} \quad P_0 = 40,000,$$

where $t_n = n$ is the number of days from the initial measurement of the population and $k(t) = 0.08 - 0.01t$ is the growth rate of this invertebrate, which is clearly declining as t increases.

a. Find the population for this organism for the first 5 days.

b. When the growth rate falls to zero, this population reaches its maximum. Find when this occurs and what the population is at that time.

c. Determine when the pollution level gets so high that this invertebrate goes extinct.

12. Many European countries are leveling off and their population will soon begin to decline as couples produce on average less than two children per couple. Italy is the slowest growing country in the world. In 1960, Italy had 50.2 million people. In 1970 and 1980, Italy had 53.7 and 56.5 million people, respectively.

a. The average growth rate for the decades listed above is 6.1% per decade. Let $P_0 = 50.2$ with $r = 0.061$ and n as the number of decades after 1960. Use the Malthusian growth model ($P_{n+1} = (1 + r)P_n$) to estimate the population of Italy in 1990 and 2000. At this growth rate, how long would it take Italy's population to double?

b. Closer examination of the data shows that the growth rate between 1960 and 1970 is 7.0%, while between 1970 and 1980 the growth rate is 5.2%. These two growth rates suggest that a declining growth rate of the form

$$k(t_n) = 3.598 - 0.0018t_n,$$

with $t_n = 1960 + 10n$. Use the Nonautonomous Malthusian Growth model

$$P_{n+1} = (1 + k(t_n))P_n,$$

with $P_0 = 50.2$ to estimate the population of Italy in 1990 and 2000. How long until this model predicts that Italy's population will level off and begin declining?

c. Census data on Italy show that its population in 1990 was 56.8 million and in 2000, it was 57.9 million. Find the percent error between the actual census data and the predictions you made in Parts a and b. Are the census data consistent with your prediction of when the Italian population will level off as computed by the Nonautonomous Malthusian Growth Model?